Oil Shocks, Capital Outflows, and Optimal Central Bank Policy*

Lakshya Narula†

November 18, 2026

Updated frequently, most recent version can be found here

Abstract

How should an economy combine interest rate and foreign exchange intervention (FXI) in response to oil price and capital-outflow shocks? I study this question in a small open economy with imported oil, foreign ownership of domestic-currency bonds, and intermediaries with limited position capacity. An oil shock alone can raise the premium intermediaries require and create a situation where FXI is valuable: lower spending and a fall in the dollar value of outstanding debt need not cover the higher import bill. A simultaneous foreign bond sale puts further pressure. I also establish when FXI improves on optimized monetary policy after accounting for financing, repayment, and inherited-debt valuation. Calibrating to Indian data, the joint cushions consumption and depreciation when the shocks coincide. Waiting for FXI for one quarter costs little when intermediaries can finance the initial trade, but is costly when their capacity binds immediately. Increasing domestic intermediary ownership reduces that loss by retaining more intermediary earnings at home, without eliminating the financing constraint.

Keywords: oil shocks; capital outflows; optimal monetary policy; exchange rate management; foreign exchange intervention; financial intermediation.

JEL codes: E52; E58; F31; F41; Q43.

*This paper is a chapter of my Ph.D. dissertation at the University of Minnesota. I thank V.V. Chari and Neil Mehrotra for their support and guidance during this project. I also thank Benjamin Malin, Thomas Tallarini, Sauhard Srivastava, Amol, other participants of the Economic Analysis Group at the Federal Reserve Bank of Minneapolis, and the Minnesota Jamboree for their comments and suggestions. All errors are my own. This is a working paper draft; comments and suggestions are welcome. First draft: September 9, 2026.

†Ph.D. candidate in Economics, University of Minnesota. Email: narul014@umn.edu.

1 Introduction

For an oil-importing economy, a rise in the world price of oil means giving up more national income to buy the same fuel. The Russia-Ukraine conflict in 2022 and the Middle East conflict in early 2026 brought a second difficulty into focus: the higher bill comes due well before many households or firms can change how much oil they use (Adrian et al., 2026). As fuel costs feed into consumer prices and the higher import bill puts pressure on the current account, the central bank must decide how much to tighten monetary policy. That choice becomes harder when foreign investors are selling domestic assets at the same time: a higher interest rate restrains spending across the economy, while a sale of foreign reserves supplies the foreign currency needed to pay for imports. When should an oil importer use both instruments, and how much does it matter whether it intervenes immediately?

India illustrates why these shocks are worth studying together. In March 2022 the price of the Indian crude basket rose by twenty percent in a month, while foreign portfolio investors sold a net half a trillion rupees of Indian securities. Four years later, in March 2026, the basket rose from $69 to $113 per barrel and net sales reached 1.26 trillion rupees. The two pressures have coincided outside these episodes as well. Between January 2013 and September 2026, the basket rose at least ten percent in fifteen months; in ten of them foreign investors were net sellers across asset classes (Figure 1).

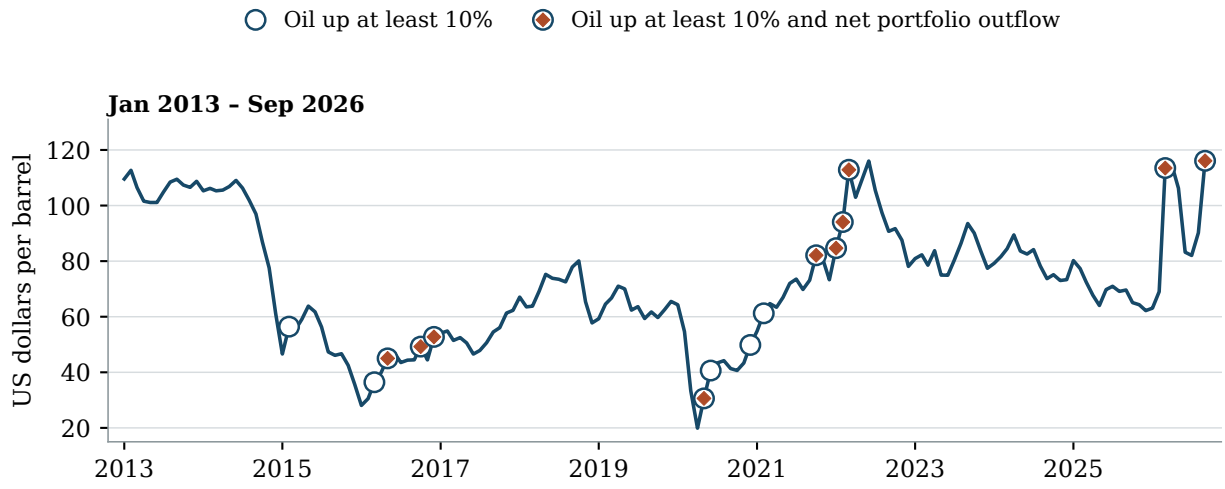


Figure 1: Oil prices and foreign portfolio outflows in India

Monthly Indian crude-basket prices, January 2013–September 2026. Circles mark price increases of at least ten percent from the previous month. Diamonds mark net foreign portfolio sales across all reported asset classes in those months. The published basket weights changed in March 2026. Sources: Petroleum Planning and Analysis Cell (2026), Central Depository Services (India) Limited (2026), and SEBI's April and October 2026 bulletins.

The two shocks meet in the domestic bond market. An importer that finances part of a higher oil bill needs a buyer for domestic debt, and a foreign investor selling domestic bonds needs a buyer as well. I study a small open economy where households consume imported oil, firms use it in production, and intermediaries obtain foreign funding to hold domestic-currency bonds. Their required return increases with their position and can rise further when they reach a position limit. Monetary policy changes the

spending and prices that determine the financing need. Sterilized foreign exchange intervention (FXI) changes the financing position directly by exchanging reserves for domestic bonds.

The currency and maturity of outstanding debt matter for that adjustment. Depreciation lowers the foreign-currency payment on a domestic-currency coupon, while the interest rate affects the price at which the remaining claim trades. An oil importer can therefore meet part of its higher bill through a reduction in foreign payments on inherited debt. The central bank takes the contractual portfolio as given and allows its current value to adjust in equilibrium. The policy question is how to combine this adjustment with household spending and the financing supplied by intermediaries and official reserves.

I begin with a complete two-period economy with preset domestic prices and flexible wages. All financial claims are settled in the second period. Monetary policy selects domestic expenditure in both periods subject to household saving and the interest-rate bound; wages, imports and portfolios satisfy private optimality and the two external budgets. Holding monetary conditions fixed, I show when a first-period oil increase raises the intermediary premium even though direct foreign investors have not reduced their bond target. The higher production oil bill induces lower import spending and revalues inherited debt, but repayment in the second period prevents those adjustments from absorbing the entire financing need. The remaining position raises the return intermediaries require. A simultaneous reduction in the foreign-investor mandate reinforces the premium response under the same monetary conditions.

The two-period economy also establishes a separate policy result. Starting from optimized monetary policy alone, a reserve sale can raise welfare without requiring the same change in domestic spending as another monetary adjustment. I derive a sufficient condition that weighs the intertemporal financing benefit and the change in future foreign payments against the valuation of inherited claims. This makes explicit why a positive premium can create a role for FXI, and why lowering the premium is not itself the policy objective. The monetary and reserve instruments affect several parts of the allocation together.

For the quantitative analysis, I extend the economy to an infinite horizon, implement monetary policy through a bounded interest-rate path, allow firms to reset prices at different times and use convex labor disutility. Energy use, financial portfolios, reserves and shock persistence are calibrated to Indian data. The financial shock captures the bond component of the portfolio sales in Figure 1: direct foreign investors reduce the foreign-currency value of domestic bonds they want to hold. The mandate path is identical across policies. Realized aggregate sales also depend on bond prices, the exchange rate and purchases by foreign-owned intermediaries, and I report those transactions separately.

I compare a Taylor rule, optimized monetary policy without FXI, and joint monetary and reserve policy. Under the oil-only shock, reserve sales allow a larger import bill and a smaller initial consumption contraction while limiting the intermediary financing required. When the oil shock coincides with the portfolio shock, joint policy reduces the first-quarter consumption fall from about one percent to 0.76% and depreciation from about 12% to 8%, relative to optimized monetary policy alone. Welfare includes both these immediate effects and the future payments and reserve income associated with the policy plan.

The final result concerns timing and ownership. A central bank required to wait one quarter can announce its future intervention immediately, but the foreign currency is unavailable for the initial trade. Waiting costs little when intermediaries can finance that trade. In the calibrated combined shock they reach their position limit immediately, and postponement sacrifices more than half of the welfare gain from adding FXI. That cost falls as the domestic ownership share rises, because more of the intermediary earnings generated by the delayed adjustment remain at home. It remains positive under full domestic ownership: the financing limit, foreign funding, direct foreign bondholders and sticky-price adjustment still matter for household welfare.

The paper proceeds as follows. Section 2 relates the analysis to work on commodity shocks and foreign exchange intervention. Sections 3 and 4 describe the two-period economy, characterize implementable allocations and state the policy problem. Section 5 establishes the oil-to-premium channel and the value of the additional intervention margin. Sections 6 and 7 introduce the infinite-horizon economy and its calibration. Section 8 presents the policy, timing and ownership comparisons, and Section 9 concludes.

2 Related literature

This paper connects the literature on monetary policy in commodity-exposed economies with the literature on foreign exchange intervention (FXI). I study an oil-importing small open economy where households consume oil, firms use it in production, and domestic prices are sticky. Because intermediaries have limited room to hold domestic-currency bonds, the central bank faces two questions when oil becomes more expensive: how much domestic spending should adjust, and how should the country finance the rest of the oil bill?

2.1 Commodity shocks and monetary policy

For an oil exporter, a fall in the oil price cuts export earnings and demand from the oil sector. Bergholt et al. (2019) trace this transmission through Norway's domestic supply chains; Omotosho (2022) studies how capital and the oil intensity of production affect the monetary response. Hevia and Nicolini (2013, 2015) examine how the exchange rate should adjust when the prices of exported and imported goods move and domestic prices or wages are slow to respond. Catão and Chang (2013) show that the ranking of monetary rules also depends on what the country imports and exports and how well it can share risk abroad.

For an importer, higher energy prices reduce purchasing power. Bodenstein et al. (2008) distinguish the response of core inflation from that of headline inflation; Bodenstein et al. (2011) show how the wealth transfer to exporters shapes trade and the real exchange rate. In Chan et al. (2024), labor and imported energy are complements. Higher energy prices reduce labor's share of income, and borrowing-constrained households cut spending. My question is how an importer finances its purchases even after spending falls.

Drechsel et al. (2019) connect commodity prices to borrowing conditions for producers in an exporting economy. Drechsel et al. (2026) extend the framework to economies that both import and

export commodities, including emerging-market importers. I follow their treatment of the imported commodity as both a consumption good and a production input. In my model, the premium on a domestic-currency bond depends on how much debt intermediaries must carry. It can rise with the oil price even if direct foreign investors keep their bond target unchanged.

Some work on commodity exporters already brings the exchange rate and reserves into the policy problem. Jin and Xiong (2021) show how fiscal stress after an oil-price fall limits exchange-rate stabilization in Russia. In Faltermeier et al. (2022), reserve accumulation during a commodity boom protects tradable production when firms learn by doing. Aktuğ and Rezghi (2026) study an exporter that receives oil income, uses oil in production, and has domestic financial intermediaries. An oil-price shock changes those intermediaries' currency exposure and gives FXI a role even without a financial shock. For the importer I study, the pressure comes from a higher oil bill. The two-period model shows when spending cuts and the lower foreign-currency value of outstanding debt leave part of that bill for intermediaries to finance.

2.2 Foreign exchange intervention

When intermediaries have limited capacity to bear currency risk, portfolio orders move exchange rates and expected returns (Gabaix and Maggiori, 2015; Maggiori, 2022). Cavallino (2019) derives the optimal intervention response to shifts in foreign demand for domestic assets. Other work identifies distinct reasons why currency markets matter for policy: the cost of sterilization under capital controls (Chang et al., 2015), banks' demand for foreign-currency liquidity at settlement (Bianchi et al., 2021), and the effect of central bank trades when investors anticipate only a limited number of rounds of market adjustment (Iovino and Sergeyev, 2023). The currency of trade invoicing also shapes exchange-rate transmission (Mukhin, 2022) and the case for floating when trade prices are set in a dominant currency (Egorov and Mukhin, 2023).

Itskhoki and Mukhin (2023) give a benchmark in which monetary policy closes the output gap and FXI removes the distortion in currency markets. In the oil-importing economy, the interest rate also affects the value of outstanding long bonds, and a reserve sale changes how much new debt intermediaries must finance. These links mean that both instruments influence what households consume and what the country later pays abroad. Fanelli and Straub (2021) show how the national cost of intervention depends on who receives the returns on currency positions, while Fanelli (2026) shows how the international portfolio determines the insurance that debt revaluation provides. The two channels enter the oil experiment through the payment due on inherited bonds and the premium earned on new ones.

Basu et al. (2025) study interest rates, FXI, capital controls and macroprudential policy together, finding a role for intervention when foreign demand shifts in shallow currency markets. In my model the available capacity of intermediaries is the additional debt they can hold before reaching their position limit. An oil shock alone can use up that capacity, and a simultaneous cut in the foreign bond target can exhaust it in the first quarter. In the Indian calibration, immediate joint intervention then improves on both the Taylor rule and the optimized interest rate alone. Increasing domestic ownership reduces the cost of waiting because more of the premium earned on the initial trade returns to domestic households.

3 Model

The economy imports oil for consumption and production, exports a contracted quantity of its domestic good, and borrows through domestic-currency bonds. Households can trade domestic claims, financial intermediaries have access to foreign funding, while a separate group of foreign investors holds domestic bonds according to an exogenous portfolio mandate. The central bank conducts monetary policy and can exchange foreign reserves for domestic bonds. Domestic ownership of intermediaries determines who receives their earnings, without changing their ability to finance a position.

We start with a two period economy. The first-period shock occurs after domestic prices have been set; policy can respond in that period. All remaining claims are paid in the second period. Domestic prices are preset at $P_{H,t} = 1$ in both periods, and wages are flexible. The foreign manufactured-good price is the numeraire, $P^*_M = 1$. The world oil price P^O_t is in foreign-good units, and S_t is domestic currency per unit of foreign currency. A rise in S_t is depreciation.

3.1 Households

A unit mass of identical households consumes the domestic good $C_{H,t}$, imported manufactures $C_{M,t}$ and imported oil $C_{E,t}$, and supplies hours N_t. Preferences are

$$\mathcal{W} = \sum_{t=0}^{1} \beta^t \left[(1-\alpha)\log C_{H,t} + \alpha(1-\omega)\log C_{M,t} + \alpha\omega \log C_{E,t} - \chi N_t\right] \tag{1}$$

Here $\beta \in (0,1)$ discounts future utility, $\alpha \in (0,1)$ is the import expenditure share, $\omega \in (0,1)$ is oil's share within imported consumption, and $\chi > 0$ scales the disutility of work. Write C_t for the corresponding unit-elastic consumption aggregate and P_t for its expenditure price index; $\log C_t$ equals the three weighted log terms up to a constant.

The household holds H_0 units of a domestic long bond priced at Q^B_0 and B^S_0 units of a one-period domestic claim priced at $Q^S_0 = 1/I_0$, where I_0 is the gross domestic interest rate between the two periods. The short claim pays one in the second period; the long claim pays L. Inherited long-bond face H_{-1} pays a coupon of one and leaves $\delta_B \in [0,1)$ surviving units. With $z_t = C_{M,t} + P^O_t C_{E,t}$ denoting import expenditure in foreign units, the two nominal budgets are

$$\begin{aligned} C_{H,0} + S_0 z_0 + Q^B_0 H_0 + Q^S_0 B^S_0 = {} & W_0 N_0 + \Pi^F_0 + (1 + \delta_B Q^B_0) H_{-1} \\ & + T_0 + J^H_0, \end{aligned} \tag{2}$$

$$C_{H,1} + S_1 z_1 = W_1 N_1 + \Pi^F_1 + L H_0 + B^S_0 + T_1 + J^H_1 \tag{3}$$

W_t is the wage, Π^F_t firm and export-contract profits, T_t the net public transfer, and J^H_t domestic intermediary income including fee rebates. The household chooses consumption, hours and its domestic portfolio subject to these budgets and $H_0 \geq 0$. It cannot invest in the foreign safe asset. All domestic claims are settled in the second period. Let ν_t be the marginal utility of domestic nominal income.

Household optimality gives

$$C_{M,t} = (1-\omega)z_t, \qquad C_{E,t} = \omega z_t / P_t^O, \tag{4}$$

$$S_t z_t = \frac{\alpha}{1-\alpha} C_{H,t}, \qquad W_t = \frac{\chi}{1-\alpha} C_{H,t}, \tag{5}$$

$$\nu_t = \frac{1-\alpha}{C_{H,t}}, \qquad \nu_0 = \beta I_0 \nu_1 \tag{6}$$

Households allocate spending across goods and across periods in the same optimization problem. The intratemporal demands determine the composition of expenditure; the Euler equation determines its intertemporal allocation.

3.2 Monetary policy

With domestic prices preset, I represent monetary policy by its control of aggregate domestic expenditure $(C_{H,0}, C_{H,1})$. This is a reduced-form policy assumption: the central bank can select these expenditure levels among allocations that satisfy household and firm optimization and all budgets. Households continue to choose their own consumption, labor and saving. The implied interest rate must satisfy

$$I_0 = \frac{C_{H,1}}{\beta C_{H,0}} \tag{7}$$

and the common rate bound stated below. Monetary policy alone and joint policy have the same expenditure choices; joint policy also chooses a reserve trade. Specifying both expenditure levels closes the final period, when there is no subsequent interest rate. The infinite-horizon model instead implements monetary policy through a sequence of interest rates and its continuation rule. The analytical comparison identifies the additional intervention margin under the stated demand-control assumption; the quantitative comparison evaluates that margin under the rate-based policy restrictions.

3.3 Firms

Firms produce the domestic good using labor and imported oil $X_{E,t}$. The oil input is distinct from household oil consumption $C_{E,t}$. Technology has constant elasticity of substitution (CES):

$$Y_t = A\left[(1-\mu)N_t^{(\nu_E-1)/\nu_E} + \mu X_{E,t}^{(\nu_E-1)/\nu_E}\right]^{\nu_E/(\nu_E-1)} \tag{8}$$

$A > 0$ is productivity, $\mu \in (0,1)$ is the oil-input weight, and $\nu_E > 0$ is the elasticity of substitution between labor and oil. At $\nu_E = 1$, the technology is $Y_t = A N_t^{1-\mu} X_{E,t}^{\mu}$. Bars denote reference values. Calibration chooses A and μ to reproduce the observed oil cost share and reference output; the CES weight need not itself equal that cost share. Export contracts specify a fixed quantity $\bar{X}$ of domestic goods and fixed foreign receipts $\bar{X}^*$. Domestic goods clearing requires $Y_t = C_{H,t} + \bar{X}$. Once prices have been set, firms meet these orders by choosing

$$\min_{N_t, X_{E,t} \geq 0} W_t N_t + S_t P_t^O X_{E,t} \quad \text{subject to (8) and} \quad Y_t \geq C_{H,t} + \bar{X} \tag{9}$$

Let mc_t denote marginal cost in units of the domestic good. Since $P_{H,t} = 1$, it is also the nominal cost of one more unit of output. Cost minimization gives

$$W_t = mc_t \frac{\partial Y_t}{\partial N_t}, \qquad S_t P_t^O = mc_t \frac{\partial Y_t}{\partial X_{E,t}} \tag{10}$$

Revenue is determined by orders at preset prices, so minimizing input costs also maximizes profits conditional on those orders. Domestic producers and export-contract holders together distribute

$$\Pi_t^F = C_{H,t} + S_t \bar{X}^* - W_t N_t - S_t P_t^O X_{E,t}$$

to households. Their profits need not be zero. Fixed export contracts remove a policy margin through export quantities and receipts, while retaining the imported-oil bill.

3.4 Financial markets

3.4.1 Domestic bonds and foreign investors

The economy inherits domestic-currency bonds. One unit pays a coupon of one in the first period and leaves $\delta_B \in [0, 1)$ units of a remaining claim. Each remaining unit pays a fixed domestic-currency amount L in the second period and expires. This is the finite-horizon counterpart of the long bond used in the infinite-horizon model. With positive household holdings, its ex-coupon price and foreign-currency returns satisfy

$$Q_0^B = \frac{L}{I_0}, \qquad R_0^B = \frac{1 + \delta_B Q_0^B}{\bar{Q}^B S_0}, \qquad R_1^B = \frac{L}{Q_0^B} \frac{S_0}{S_1} = I_0 \frac{S_0}{S_1} \tag{11}$$

The opening exchange rate is one. The foreign gross safe return is constant at $r = 1/\beta$, and the reference contract satisfies $L = r\bar{Q}^B = 1 + \delta_B \bar{Q}^B$. Inherited face values are given before policy is chosen. Their current value is determined by S_0 and Q_0^B; neither is fixed by an insurance payment. Taking the inherited portfolio into the policy problem in this way follows Fanelli and Straub (2021); Fanelli (2026).

Financial positions and net exports are measured in foreign-good units. Let B_t^F denote the domestic-currency face held by direct foreign investors. Their first-period mandate is

$$F_0 = \frac{Q_0^B B_0^F}{S_0}, \qquad F_0 \leq \bar{F} \tag{12}$$

An investor shock reduces F_0, the foreign-currency value investors wish to hold. Oil-only experiments set $F_0 = \bar{F}$. The mandate distinguishes a shift in foreign bond demand from the subsequent transactions that achieve it: bond prices and the exchange rate determine the face value bought or sold. It is the mandate, not aggregate realized sales by all nonresidents, that is held fixed across policy comparisons.

3.4.2 Intermediaries and ownership

A unit mass of price-taking intermediaries finances domestic bonds in the foreign safe market. Domestic households own a share $s \in [0, 1]$, and foreign households own the remainder. Each intermediary faces the same prices, marginal fee and position limit; with unit mass its position D_0 is also the aggregate

position. The ownership split therefore leaves the private bond-demand schedule unchanged, as in the ownership accounting of Cavallino (2019).

Define the premium on a new position as $\psi_0 = R_1^B/r - 1$. Each intermediary solves

$$\max_{D_0} \ \psi_0(D_0 - \bar{D}) - \frac{\gamma}{2}(D_0 - \bar{D})^2 \quad \text{subject to} \quad -D^{\max} \leq D_0 \leq D^{\max} \tag{13}$$

Here $\bar{D} > 0$ is its normal position, $\gamma > 0$ is the marginal-fee slope, and $D^{\max} = \ell K$ is its position limit. K is usable intermediary capital and ℓ is exposure allowed per unit of capital. Owners receive gains and cover losses while maintaining usable capital. A negative position is a short sale. Subtracting $\psi_0 \bar{D}$ from gross earnings does not affect the choice because prices and $\bar{D}$ are taken as given.

The quadratic charge is a private fee. The domestic fee receipts are rebated lump sum to domestic households, which also own the domestic intermediaries. Thus net distributions and the rebate add to gross domestic intermediary income; the fee is not a resource loss or a separate term in national welfare. A domestic intermediary nevertheless pays the marginal charge when it changes its own position, taking the rebate as given. This preserves a private financing wedge even when the transfer remains within the economy. Foreign-owned intermediaries' payments are external payments. The distinction between national payments and intermediary incentives is central to the welfare accounting in Fanelli and Straub (2021).

Let $\lambda_0^+, \lambda_0^- \geq 0$ be the multipliers on the two position limits. The strictly concave problem gives

$$\begin{aligned} \psi_0 &= \gamma(D_0 - \bar{D}) + \lambda_0^+ - \lambda_0^-, \\ \lambda_0^+(D^{\max} - D_0) = 0, &\qquad \lambda_0^-(D^{\max} + D_0) = 0 \end{aligned} \tag{14}$$

Within capacity, the premium increases with the position. At the upper limit, further financing pressure must be absorbed by prices and other portfolios. The extra premium then includes the value of scarce intermediary capacity.

3.5 Market clearing and the external budget

Let R_t be official foreign reserves and $\bar{G}$ a predetermined nonreserve foreign position. The latter represents an inherited balance-sheet item, not a foreign asset that households can freely choose. Let B_{-1}^G be inherited government bond face and B_t^{CB} the face held by the central bank. A reserve sale is a purchase of domestic bonds:

$$R_0 = R_{-1} - Z_0, \qquad Q_0^B(B_0^{CB} - \delta_B B_{-1}^{CB}) = S_0 Z_0 \tag{15}$$

The central bank's earnings are remitted to the fiscal authority. Writing this remittance as Π_t^{CB}, its cash budgets are

$$\Pi_0^{CB} + Q_0^B(B_0^{CB} - \delta_B B_{-1}^{CB}) + S_0 R_0 = B_{-1}^{CB} + S_0 r R_{-1}, \tag{16}$$

$$\Pi_1^{CB} = L B_0^{CB} + S_1 r R_0 \tag{17}$$

A negative remittance is a fiscal transfer to the central bank. With the predetermined position $\bar{G}$ included in the public account, the fiscal budgets are

$$T_0 + B^G_{-1} = \Pi^{CB}_0 + S_0(r-1)\bar{G}, \tag{18}$$

$$T_1 + \delta_B L B^G_{-1} = \Pi^{CB}_1 + S_1 r \bar{G} \tag{19}$$

Lump-sum transfers adjust to satisfy these budgets. In the final period the public sector redeems every remaining bond and settles its foreign assets. Domestic intermediary distributions, including the fee rebate, are

$$J^H_0 = sS_0(R^B_0 - r)D_{-1}, \qquad J^H_1 = sS_1(R^B_1 - r)D_0 \tag{20}$$

Goods and domestic bonds clear according to

$$Y_t = C_{H,t} + \bar{X}, \qquad B^S_0 = 0, \tag{21}$$

$$B^G_{-1} = H_{-1} + B^{CB}_{-1} + \frac{F_{-1} + D_{-1}}{\bar{Q}^B}, \tag{22}$$

$$\delta_B B^G_{-1} = H_0 + B^{CB}_0 + \frac{S_0}{Q^B_0}(F_0 + D_0) \tag{23}$$

The common N_t is both household labor supplied and firms' labor input. Imported manufactures equal $C_{M,t}$, and total oil imports equal $C_{E,t} + X_{E,t}$; their foreign-currency payments enter net exports. Foreign lenders supply the intermediaries' funding at r. The remaining foreign-exchange clearing condition is the external budget below.

Define net exports and the foreign-owned domestic bond position by

$$NX_t = \bar{X}^* - z_t - P^O_t X_{E,t}, \qquad V_t = F_t + (1-s)D_t \tag{24}$$

National net foreign assets before the shock and after first-period trading are

$$A_{-1} = R_{-1} + \bar{G} - F_{-1} - D_{-1}, \qquad A_0 = R_0 + \bar{G} - F_0 - D_0 \tag{25}$$

The whole intermediary position enters the second expression. Domestic intermediaries borrow abroad to fund their domestic bonds, so changing the owners does not make that funding a domestic resource. Ownership instead determines how much of the excess bond return is paid abroad. Aggregating the private and public budgets gives

$$A_0 = rA_{-1} + NX_0 - (R^B_0 - r)V_{-1}, \tag{26}$$

$$0 = rA_0 + NX_1 - r\psi_0 V_0 \tag{27}$$

The first equation finances current trade and the return on inherited claims. The second settles every remaining claim, including the reserve position, after second-period trade. It imposes solvency at the end of a finite economy; it does not impose reference consumption or employment. There is no second-period intervention decision hidden in this settlement.

3.6 Competitive equilibrium

Definition 1 (Competitive equilibrium)**.** Given inherited portfolios $(H_{-1}, B^G_{-1}, B^{CB}_{-1}, F_{-1}, D_{-1}, R_{-1}, \bar{G})$, the contract (δ_B, L) and opening bond price $\bar{Q}^B$, preset prices $P_{H,t} = 1$, shocks (P^O_0, P^O_1, F_0), and monetary expenditure settings $(C_{H,0}, C_{H,1})$ with reserve trade Z_0, a competitive equilibrium consists of allocations $\{C_{H,t}, C_{M,t}, C_{E,t}, N_t, X_{E,t}, Y_t\}^1_{t=0}$, prices $\{S_t, W_t, P_t, mc_t\}^1_{t=0}$, portfolios $(B^S_0, H_0, B^{CB}_0, B^F_0, D_0, R_0)$, transfers and income $\{T_t, J^H_t, \Pi^F_t, \Pi^{CB}_t\}^1_{t=0}$, and bond prices (Q^S_0, Q^B_0) such that households maximize (1) subject to (2)–(3); aggregate domestic expenditure equals its monetary settings; firms solve (9); foreign positions and intermediary choices satisfy (12)–(14); and public budgets and markets satisfy (15)–(27). Consumption is positive, household and central-bank bond holdings are nonnegative, and policy respects (29). Returns and aggregate foreign positions follow their definitions above.

4 Equilibrium characterization and policy

4.1 Implementable allocations

The central bank operates through monetary conditions and reserve trades and these choices induce household spending, production and financial positions through private optimization. An allocation is implementable if there is an admissible policy under which households, firms and intermediaries choose it and all budgets clear. This characterization lets the planner choose allocations subject to their equilibrium restrictions, rather than repeatedly attaching a separate policy calculation to each allocation.

Two wedges organize the policy problem. A wedge is a difference between the household's marginal valuation and the marginal cost of obtaining a good or financing it. With the domestic price fixed at one, the *domestic wedge* is $1 - mc_t$. An extra unit of domestic consumption is worth ν_t in utility, while the labor and oil needed to produce it cost $\nu_t mc_t$. A positive wedge makes additional domestic consumption valuable before accounting for the foreign resources needed to support it.

The *financial wedge* concerns expenditure across periods. Define $\lambda_t = S_t \nu_t = \alpha / z_t$ as the marginal utility of foreign expenditure. Combining household saving, the exchange-rate conversion of bond returns and $\beta r = 1$ gives

$$1 + \psi_0 = \frac{z_1}{z_0} \tag{28}$$

Equivalently, $\lambda_0 - \beta r \lambda_1 = \lambda_1 \psi_0$. With a positive premium, an additional unit of foreign expenditure in the first period is worth more than its safe-rate repayment costs in discounted second-period utility. Households cannot obtain that financing directly because they cannot buy or borrow through the foreign safe asset. Intermediaries supply it subject to their marginal fee and position limit. These two wedges identify the margins policy can improve while repayment and inherited-debt valuation determine the cost of doing so.

For any positive set of allocations $(C_{H,0}, C_{H,1}, z_0, z_1, D_0)$, household demand pins down the exchange rates and wages, production pins down the labor and oil inputs, and saving pins down I_0. The first external budget then determines the reserve sale that would be required and the second external budget is an additional restriction of why financing today must be paid for tomorrow. The allocations must

also satisfy intermediary optimality and the operational bounds:

$$I_0 \geq 1, \qquad |\log(I_0/r)| \leq \overline{\Delta i}, \qquad |Z_0| \leq \bar{Z}, \qquad \underline{R} \leq R_{-1} - Z_0 \leq R_{-1},$$
$$|D_0| \leq D^{\max}, \qquad H_0 \geq 0, \qquad B_0^{CB} \geq 0 \tag{29}$$

H_0 and B_0^{CB} are household and central-bank bond faces. The constants $\bar{Z}$, $\underline{R}$ and $\overline{\Delta i}$ are the intervention-flow ceiling, reserve floor and maximum log rate change. The opening rate is r. These bounds apply to both policy choices.

Proposition 1 (Implementability conditions)**.** *Under the monetary demand-control assumption, consider positive allocations with positive household bond holdings. They are competitive-equilibrium allocations if and only if they satisfy household demands and labor optimality, cost minimization and goods clearing,* (28)*, intermediary optimality, both external budgets, and the portfolio and policy bounds. Monetary policy selects their domestic expenditure levels, with*

$$I_0 = \frac{C_{H,1}}{\beta C_{H,0}}, \qquad Z_0 = R_{-1} + \bar{G} - F_0 - D_0 - A_0$$

and transfers determined by the public budgets.

Appendix B proves both directions and constructs the nominal accounts. Positive household bond holdings allow the short and long domestic returns to be equated. All reported quantitative allocations also satisfy that portfolio condition.

The characterization retains inherited-debt valuation. For inherited foreign-held face B_{-1}^{ext},

$$R_0^B V_{-1} = \frac{(1 + \delta_B Q_0^B) B_{-1}^{ext}}{S_0} \tag{30}$$

The coupon and face are contractual; their current foreign value responds to policy through the bond price and exchange rate. Taking these payments into the implementable set follows the portfolio approach in Fanelli and Straub (2021); Fanelli (2026). It is particularly important for an importer: a smaller foreign payment on inherited debt can finance part of the higher oil bill.

4.2 Planning problem

Let $\mathcal{E}(P_0^O, P_1^O, F_0)$ be the implementable set in Proposition 1, conditional on inherited portfolios. The central bank chooses the allocation with the highest household welfare in that set:

$$\begin{aligned} \max_{\{C_{H,t}, z_t, N_t, X_{E,t}\}_{t=0}^1, D_0, Z_0} \quad & \sum_{t=0}^{1} \beta^t \left[(1-\alpha) \log C_{H,t} + \alpha \log z_t - \alpha\omega \log P_t^O - \chi N_t \right] \\ \text{subject to} \quad & \{C_{H,t}, z_t, N_t, X_{E,t}, D_0, Z_0\} \in \mathcal{E}(P_0^O, P_1^O, F_0) \end{aligned} \tag{31}$$

Terms constant across policies have been omitted from utility. In particular, membership in $\mathcal{E}$ requires

$$\begin{aligned} Y_t &= C_{H,t} + \bar{X}, & W_t &= mc_t \frac{\partial Y_t}{\partial N_t}, \\ S_t P_t^O &= mc_t \frac{\partial Y_t}{\partial X_{E,t}}, & z_1 &= (1+\psi_0) z_0, \\ A_0 &= R_{-1} - Z_0 + \bar{G} - F_0 - D_0, & A_0 &= rA_{-1} + NX_0 - (R_0^B - r)V_{-1}, \\ 0 &= rA_0 + NX_1 - r\psi_0 V_0 \end{aligned}$$

together with production (8), household conditions (4)–(6), bond returns (11), intermediary conditions (14), and the bounds (29). Household and public budgets and bond clearing remain (2)–(3) and (15)–(23). These restrictions preserve household and firm optimization when the planner chooses an allocation: the expenditure settings must be consistent with private choices, financing and repayment.

Monetary policy alone adds $Z_0 = 0$ as an additional constraint. *Joint policy* chooses Z_0 under the stated bounds. Both optimize monetary policy in both periods, face the same shock and inherit the same contracts. To establish a strict benefit from FXI, however, one must find a reserve trade that improves household welfare after private behavior, repayment and inherited valuations adjust. The next section supplies such a test.

5 Discussion

5.1 Oil and the intermediary premium

Consider a rise in the first-period oil price with $P_1^O = \bar{P}^O$ and no change in the foreign-investor mandate. Hold the monetary expenditure settings and intervention fixed to isolate how the oil bill passes through the financial market. This means that the domestic spending $C_{H,0}, C_{H,1}$ and the rate I_0 then remain fixed while import spending, production inputs, exchange rates, and payments on inherited bonds adjust in equilibrium.

This comparison describes transmission under a specified monetary response but it does not require the optimizing central bank to retain that response. Now, differentiating each period's budget along this oil experiment gives

$$dD_0 = -dNX_0 + V_{-1}\, dR_0^B, \tag{32}$$

$$dNX_1 = r[1 + (1-s)\psi_0]\, dD_0 + rV_0\, d\psi_0 \tag{33}$$

The first equation says how much financing remains after trade and inherited payments adjust. The second says how much net export revenue is needed to repay it. Household saving connects the two periods through

$$dz_1 = (1+\psi_0)\, dz_0 + z_0\, d\psi_0 \tag{34}$$

An increase in the premium therefore changes the relative amounts households can spend abroad across the two periods.

Proposition 2 (Oil and the premium)**.** *Suppose $0 < \nu_E \leq 1$, foreign-owned domestic claims are nonnegative in both periods, and gross returns are positive. At a regular equilibrium, a small first-period oil-price increase raises the intermediary premium when the monetary path and intervention are fixed. Within the position limits,*

$$\frac{dD_0}{dP_0^O} > 0, \qquad \frac{d\psi_0}{dP_0^O} > 0$$

At a strictly binding upper limit the position remains fixed and the premium rises. A simultaneous decrease in F_0 reinforces the local premium response under the same monetary conditions. Starting from a zero premium, an oil shock alone can therefore generate a positive premium without a foreign-investor withdrawal.

The production condition matters for the sign because with limited substitution between labor and oil, higher oil prices increase the production oil bill at unchanged foreign household spending. Households adjust by reducing imports, and the resulting exchange-rate change lowers the foreign payment on inherited domestic-currency debt. Repayment in the second period limits how far these adjustments can absorb the bill. Solving (32)–(34) together gives a larger intermediary position and a higher premium inside capacity; at the upper limit, the position cannot rise and the scarcity premium increases instead. Appendix C derives the closed-form derivative and each sign.

For the oil-only comparison, import spending falls in the first period and weakly in the second. The second-period fall is strict inside capacity; at a binding upper limit it is strict when $V_0 > 0$. Domestic-good spending is fixed by the maintained monetary path, so the result concerns the composition of consumption, production inputs and external financing. A withdrawal puts additional pressure on intermediary financing, while also reducing the direct foreign claim to be repaid. Appendix C retains both effects and shows that the premium response remains positive.

The combined shock's import-spending responses depend on their balance. The mandate also does not prohibit foreign-investor transactions. Keeping F_0 unchanged when the foreign value of inherited bonds falls requires investors to replace part of that lost value with purchases. Those transactions are already included in the external budgets. Proposition 2 establishes that, even after that response, the remaining financing need can raise the intermediary premium.

5.2 What FXI adds to monetary policy

Monetary policy influences the premium by changing domestic demand, input use, the exchange rate and bond valuation. Moving all these objects together creates a trade-off: a monetary adjustment that makes foreign financing easier may also move domestic consumption and employment away from the allocation the household prefers. FXI provides a different way to change the financing position. A reserve sale supplies foreign currency and transfers domestic bonds to the central bank, without requiring the same initial movement in domestic spending.

The two wedges make this trade-off precise. Recall that the marginal utility of foreign expenditure is

$$\lambda_t = S_t \nu_t = \frac{\alpha}{z_t} \tag{35}$$

At fixed oil prices and investor mandates, cost minimization and household optimality give the following welfare change along any feasible policy adjustment:

$$d\mathcal{W} = \sum_{t=0}^{1} \beta^t \nu_t (1 - mc_t)\, dC_{H,t} + \lambda_1 \psi_0 (dZ_0 + s\, dD_0) - \lambda_1 V_0\, d\psi_0 - \lambda_0 V_{-1}\, dR_0^B$$

The first term values changes in domestic consumption at the domestic wedge. The second values changes in the timing of foreign expenditure at the financial wedge. The last two terms account for changes in future foreign payments and the value of inherited debt. Monetary policy moves domestic spending and financing together. FXI provides another feasible adjustment, whose value depends on the sum of these effects. Appendix D derives the identity from production and both external budgets.

At a monetary-only optimum, hold its two monetary settings fixed and consider a small feasible increase in Z_0. Domestic consumption then remains fixed in both periods, so the first term vanishes. Use a derivative subscript Z to denote the response to that reserve sale. Its marginal welfare effect is

$$\mathcal{W}_Z = \underbrace{\lambda_1 \psi_0 (1 + s D_{0,Z})}_{\text{timing of resources}} - \underbrace{\lambda_1 V_0 \psi_{0,Z}}_{\text{future foreign payments}} - \underbrace{\lambda_0 V_{-1} R^B_{0,Z}}_{\text{inherited payments}} \tag{36}$$

The first term values the change in when foreign resources are available, given the financial wedge. The second records how intervention changes the return paid on claims held abroad. The third includes the revaluation of debt inherited before the shock. Under this fixed-monetary-path test, $R^B_{0,Z} = (R^B_0/z_0) z_{0,Z}$. An appreciation raises the foreign value of that inherited obligation and offsets part of an intervention's benefit.

The household and production conditions imply $\partial NX_t/\partial z_t < 0$: increasing import spending worsens net exports. This derivative includes the associated response of oil used in production. The following sufficient condition combines these trade responses and foreign payments, without treating a lower premium as an objective in itself.

Proposition 3 (Value of intervention)**.** *Consider a regular monetary-only optimum with room for a small reserve sale, $\psi_0 \geq 0$, $V_0 \geq 0$ and $V_{-1} \geq 0$. Suppose intermediary holdings are interior or strictly at the upper limit. Joint policy strictly improves on monetary policy alone if*

$$(1 + \psi_0) V_0 \left(-r \frac{\partial NX_0}{\partial z_0} - \frac{\partial NX_1}{\partial z_1} \right) + \psi_0 z_0 \frac{\partial NX_0}{\partial z_0} \frac{\partial NX_1}{\partial z_1} > -\frac{\partial NX_1}{\partial z_1} V_{-1} R^B_0 \tag{37}$$

The derivatives hold the monetary settings and oil prices fixed. The condition applies to an oil-only shock and to a combined oil and investor-mandate shock.

The first two terms in (37) capture the financing and future-payment benefits of changing the intermediary position and return. The right-hand side captures the inherited foreign claim that is revalued. Under the condition, the net derivative in (36) is positive. A reserve sale then raises welfare while keeping the optimized monetary path unchanged; allowing the central bank to adjust that path as well cannot reduce the best attainable welfare. Appendix D derives the exact derivatives both inside capacity and at the upper limit.

The investor shock changes this comparison through two channels. It reduces the direct foreign position F_0, and the additional financing required can push intermediaries against their limit. With a binding limit, intervention lowers the scarcity premium even though intermediaries cannot reduce their position immediately. A smaller F_0 also changes the foreign claim receiving the next return. These effects need not be proportional to the size of the mandate reduction, which is why the combined experiment must solve the same economy rather than add separate oil and portfolio responses.

5.3 Ownership, private wedges and national welfare

At $s = 1$, intermediary earnings and fee receipts stay at home. Their private pricing condition and their capacity limit remain, and direct foreign investors still hold bonds. Consequently $V_0 = F_0$ need not be zero, and the financial wedge in (28) can remain positive. FXI may improve the timing and composition of household consumption or reduce payments to foreign investors even though it saves no domestic resource cost from the fee itself.

Both instruments affect the two wedges: intervention changes import spending and inherited valuations, while monetary policy changes the financing requirement as well as domestic demand. The planner weighs their combined effect on consumption and hours under the policy and portfolio restrictions. Neither $mc_t = 1$ nor $\psi_0 = 0$ is imposed as an additional target, and the optimal premium can have either sign. The clean separation of monetary and financial stabilization in Itskhoki and Mukhin (2023) provides a benchmark for these remaining trade-offs.

6 Extended Model and Quantitative Results

The infinite-horizon economy preserves imported oil, flexible wages, the investor mandate, intermediary ownership and fees, position limits, and fixed export contracts. It replaces terminal settlement with a long coupon bond, permits staggered price adjustment, and uses convex labor disutility. Household consumption is written as a CES aggregate; its calibrated unit elasticities reproduce the log expenditure shares of the two-period model. Production retains the same CES elasticity. The reference balance sheet is now calibrated to India rather than liquidated after two periods.

The analytical economy establishes a financing mechanism and a sufficient condition for a useful second instrument. In the infinite-horizon economy, today's policy also changes future price dispersion and the assets carried into later quarters. Those effects enter both the equilibrium and welfare below. Monetary policy now chooses the interest-rate path under the infinite-horizon equilibrium and policy rule. The welfare decomposition carries over, although the finite model's direct expenditure controls and the quantitative rate controls define different policy sets. All wages remain flexible.

6.1 Preferences, goods, and household saving

The household chooses $\{C_{H,t}, C_{M,t}, C_{E,t}, N_t, B_t^S, H_t\}_{t\geq 0}$ to maximize

$$\sum_{t\geq 0} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi_N \frac{N_t^{1+\varphi}}{1+\varphi} \right], \qquad \sigma > 0, \quad \varphi \geq 0 \tag{38}$$

Write $u(C_t)$ for the first term, with its continuous limit $\log C_t$ when $\sigma = 1$. The coefficient σ is inverse intertemporal substitution elasticity, φ is inverse Frisch labor-supply elasticity, and $\chi_N > 0$ sets the labor scale. For a given reference labor supply, set $\chi_N = \bar{v}\bar{W}/\bar{N}^{\varphi}$ to preserve its labor condition as curvature changes. Let $C_{F,t}$ denote the imported-consumption basket. Consumption uses the nested CES aggregates

$$\begin{aligned} C_t &= \left[(1-\alpha)^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \alpha^{1/\eta} C_{F,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \\ C_{F,t} &= \left[(1-\omega)^{1/\vartheta} C_{M,t}^{(\vartheta-1)/\vartheta} + \omega^{1/\vartheta} C_{E,t}^{(\vartheta-1)/\vartheta}\right]^{\vartheta/(\vartheta-1)} \end{aligned} \tag{39}$$

The elasticities $\eta > 0$ and $\vartheta > 0$ govern substitution between domestic and imported consumption and between imported manufactures and energy, respectively. Use the continuous Cobb–Douglas limits at unit elasticity, including their normalization constants. The domestic-currency prices of imported manufactures and oil are $P_{M,t} = S_t$ and $P_{E,t} = S_t P_t^O$, and $P_{F,t}$ is the price index of the imported basket. With the same domestic bonds, the household budget is

$$\begin{aligned} P_t C_t + Q_t^S B_t^S + Q_t^B H_t = W_t N_t + B_{t-1}^S + (1 + \delta_B Q_t^B) H_{t-1} + \Pi_t^F + T_t \\ + S_t[(r-1)\bar{G} + s(R_t^B - r) D_{t-1}] \end{aligned} \tag{40}$$

The last term is predetermined foreign income plus domestic intermediary income, including fee rebates. The fixed position $\bar{G}$ is rolled over and is not chosen by the household. Profits Π_t^F include export-contract income and are net of the lump-sum financing of the price-setting subsidy below. The household faces nonnegative long-bond holdings and the usual no-Ponzi and transversality conditions. Appendix E derives its conditions from this budget.

Let ν_t be the utility value of a domestic-currency unit and ζ_t^H the multiplier on the nonnegative long-bond holding. The price indices, demands and saving conditions are

$$\begin{aligned} P_{F,t} &= S_t[(1-\omega) + \omega (P_t^O)^{1-\vartheta}]^{1/(1-\vartheta)}, \\ P_t &= [(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{1/(1-\eta)}, \end{aligned} \tag{41}$$

$$C_{H,t} = (1-\alpha)(P_{H,t}/P_t)^{-\eta} C_t, \qquad C_{F,t} = \alpha (P_{F,t}/P_t)^{-\eta} C_t, \tag{42}$$

$$\begin{aligned} \nu_t &= C_t^{-\sigma}/P_t, & \chi_N N_t^{\varphi} &= \nu_t W_t, \\ \nu_t Q_t^S &= \beta \nu_{t+1}, & \nu_t Q_t^B &= \beta \nu_{t+1}(1 + \delta_B Q_{t+1}^B) + \zeta_t^H, \\ \zeta_t^H &\geq 0, & H_t &\geq 0, & \zeta_t^H H_t &= 0 \end{aligned} \tag{43}$$

Within-import demands follow the same formula with shares $1-\omega, \omega$ and elasticity ϑ; Appendix E derives each demand. The same marginal utility of income governs spending and saving. On an interior portfolio the one-quarter domestic returns on the short claim and the long bond coincide, so the long-bond pricing formula in (47) continues to apply. The inequality in (43) covers a zero long holding.

6.2 Production and price adjustment

A domestic variety $j \in [0,1]$, sold at price $P_{H,t}(j)$, uses the same technology:

$$Y_t(j) = A\left[(1-\mu)N_t(j)^{(\nu_E-1)/\nu_E} + \mu X_{E,t}(j)^{(\nu_E-1)/\nu_E}\right]^{\nu_E/(\nu_E-1)} \tag{44}$$

As before, mc_t is marginal cost in domestic-good units; nominal unit cost is $P_{H,t}mc_t$. Common factor prices imply the same input ratio at every firm. Cost minimization gives

$$\begin{aligned} \frac{X_{E,t}}{N_t} &= \left[\frac{\mu}{1-\mu}\frac{W_t}{P_{E,t}}\right]^{\nu_E}, \\ P_{H,t}mc_t &= \frac{1}{A}\left[(1-\mu)^{\nu_E}W_t^{1-\nu_E} + \mu^{\nu_E}P_{E,t}^{1-\nu_E}\right]^{1/(1-\nu_E)} \end{aligned} \tag{45}$$

Domestic varieties have substitution elasticity $\epsilon_p > 1$. Under Calvo (1983) price contracts, a fraction $1-\theta_p$ can reset its price each period; the others retain their nominal price. There is no price indexation. The fixed revenue subsidy satisfies $1+\bar{\varsigma}^p = \epsilon_p/(\epsilon_p-1)$, so it corrects the steady monopoly markup and stays fixed under every policy.

Write $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ for gross domestic-good inflation and $P_{H,t}^{\#}$ for the newly chosen price. Then $p_t^{\#} = P_{H,t}^{\#}/P_{H,t}$ is the reset price relative to the domestic price index. The aggregate price index and price dispersion satisfy

$$1 = \theta_p \Pi_{H,t}^{\epsilon_p - 1} + (1-\theta_p)(p_t^{\#})^{1-\epsilon_p}, \qquad \Delta_t^p = \int_0^1 [P_{H,t}(j)/P_{H,t}]^{-\epsilon_p}\, dj$$

A resetting firm maximizes the discounted profits earned while its price remains in place. Appendix E derives its optimal reset price and the full recursions (94). Initial prices are symmetric, $\Delta_{-1}^p = 1$. Aggregation and goods clearing require

$$\Delta_t^p Y_t = A\left[(1-\mu)N_t{}^{(\nu_E-1)/\nu_E} + \mu X_{E,t}{}^{(\nu_E-1)/\nu_E}\right]^{\nu_E/(\nu_E-1)}, \qquad Y_t = C_{H,t} + \bar{X} \tag{46}$$

The fixed export contract delivers $\bar{X}$ domestic goods for $\bar{X}^*$ foreign units. Its profit is $S_t\bar{X}^* - P_{H,t}\bar{X}$. Production oil and household oil remain separate quantities. Flexible wages imply neither a wage Phillips curve nor a wage-dispersion state.

6.3 Portfolios and the policy problem

At the end of quarter t, each long bond pays one unit of domestic currency next quarter and leaves δ_B surviving units. Positive household holdings imply

$$I_t Q_t^B = 1 + \delta_B Q_{t+1}^B, \qquad Q_t^B = \sum_{j=1}^{\infty} \frac{\delta_B^{j-1}}{I_t I_{t+1} \cdots I_{t+j-1}}, \tag{47}$$

where the second expression uses the no-bubble condition. The return and premium are

$$R_t^B = \frac{1+\delta_B Q_t^B}{Q_{t-1}^B}\frac{S_{t-1}}{S_t}, \qquad \psi_t = \frac{R_{t+1}^B}{r} - 1 \tag{48}$$

The same intermediary problem applies in every quarter, with current position D_t, normal position $\bar{D}$, and unchanged limit $D^{\max}$. Its domestic fee receipts are rebated. The direct foreign-investor target and the oil price follow the specified MIT paths in Section 7.

The consolidated external accounts are

$$A_t = R_t + \bar{G} - F_t - D_t, \qquad V_t = F_t + (1-s)D_t, \tag{49}$$

$$A_t = rA_{t-1} + NX_t - (R_t^B - r)V_{t-1}, \qquad R_t = R_{t-1} - Z_t, \tag{50}$$

$$NX_t = \bar{X}^* - C_{M,t} - P_t^O(C_{E,t} + X_{E,t})$$

The predetermined foreign position $\bar{G}$ pays its interest income and is rolled over; it is not a freely chosen household asset. Domestic intermediary funding and ownership aggregate as in the finite economy. The fiscal authority maintains a prescribed real gross face $B_t^G/P_t = \bar{b}^G$. Central-bank holdings and market clearing obey

$$\frac{B_t^{CB}}{P_t} = \frac{B_{t-1}^{CB}}{P_{t-1}} + \frac{S_t Z_t}{P_t Q_t^B}, \qquad B_t^G = H_t + B_t^{CB} + \frac{S_t}{Q_t^B}(F_t + D_t) \tag{51}$$

The central bank's and fiscal authority's cash budgets are

$$\Pi_t^{CB} + Q_t^B B_t^{CB} + S_t R_t = (1 + \delta_B Q_t^B) B_{t-1}^{CB} + S_t r R_{t-1}, \tag{52}$$

$$T_t + (1 + \delta_B Q_t^B) B_{t-1}^G = Q_t^B B_t^G + \Pi_t^{CB} \tag{53}$$

As in the two-period economy, Π_t^{CB} is the central-bank remittance and T_t the net transfer to households. Adding the two budgets eliminates the remittance:

$$T_t = Q_t^B(B_t^G - B_t^{CB}) - (1 + \delta_B Q_t^B)(B_{t-1}^G - B_{t-1}^{CB}) - S_t(R_t - rR_{t-1}) \tag{54}$$

The predetermined income from $\bar{G}$ enters the household budget directly in this infinite-horizon economy. Assigning it to the fiscal transfer instead gives the same consolidated allocation. These accounting conventions are common to every policy.

Goods clearing is (46), domestic bond clearing is (51), and the short domestic claim has zero net supply, $B_t^S = 0$. Labor and production-oil markets satisfy

$$N_t = \int_0^1 N_t(j)\, dj, \qquad X_{E,t} = \int_0^1 X_{E,t}(j)\, dj$$

Households supply the common N_t. Imports are $C_{M,t}$ manufactures and $C_{E,t} + X_{E,t}$ oil. Varieties clear at $Y_t(j) = [P_{H,t}(j)/P_{H,t}]^{-\epsilon_p} Y_t$, foreign investors hold $B_t^F = S_t F_t / Q_t^B$, and intermediary bond face is $B_t^D = S_t D_t / Q_t^B$. Together with foreign funding at r, the external budget clears the foreign-exchange market.

The financial wedge also survives the extension. With $\lambda_t = S_t \nu_t = S_t C_t^{-\sigma}/P_t$, household saving gives

$$\lambda_t = \beta r (1 + \psi_t) \lambda_{t+1} \tag{55}$$

Under the calibrated $\sigma = 1$, household saving gives $I_t = P_{t+1} C_{t+1}/(\beta P_t C_t)$. The central bank chooses the admissible rate path, and spending follows from the full equilibrium and its continuation. This Euler

relation constrains expenditure growth; it does not give policy an independently chosen expenditure level in every quarter. The analytical demand-control assumption is therefore replaced by the rate implementation in the infinite-horizon economy.

Let x_t collect equilibrium prices, quantities and portfolios. After the shock, the central bank commits to rates and reserve trades for $W = 24$ quarters. It solves

$$\begin{aligned}
\max_{\{I_t, Z_t\}_{t=0}^{W-1}, \{x_t\}_{t\geq 0}} \quad & \sum_{t\geq 0} \beta^t \left[u(C_t) - \chi_N \frac{N_t^{1+\varphi}}{1+\varphi} \right] \\
\text{subject to} \quad & (40)\text{–}(46), (47)\text{–}(54), \text{ and } (94), \\
& \psi_t = \gamma(D_t - \bar{D}) + \lambda_t^+ - \lambda_t^-, \quad |D_t| \leq D^{\max}, \\
& \lambda_t^{\pm} \geq 0, \quad \lambda_t^+ (D^{\max} - D_t) = \lambda_t^- (D^{\max} + D_t) = 0, \\
& H_t \geq 0, \quad B_t^{CB} \geq 0, \\
0 \leq t < W: \quad & I_t \geq 1, \quad |\log I_t - \log I_{t-1}| \leq \overline{\Delta i}, \\
& |Z_t| \leq \bar{Z}, \quad \underline{R} \leq R_t \leq R_{-1}, \\
t \geq W: \quad & Z_t = 0, \quad I_t = I_t^{\text{Taylor}}
\end{aligned} \tag{56}$$

The household solvency and transversality conditions continue to apply. The Taylor continuation is specified in (58), with the portfolios and prices produced by the policy plan, including all previous reserve sales. Monetary policy alone adds $Z_t = 0$ in every quarter. Joint policy can use both instruments; delayed joint policy additionally imposes $Z_0 = 0$ while still allowing the interest rate to respond immediately.

I solve the nonlinear equilibrium and evaluate household utility along its paths. The position limit can bind in some quarters and not others, so the return paid on additional financing changes across the adjustment. A single linear solution around the zero-premium reference would omit that distinction. The computational approximation is confined to the distant continuation, with the actual accumulated portfolios carried into it; Appendix G reports the solution and horizon checks.

The two wedges also carry over. Let $u_t = u(C_t) - \chi_N N_t^{1+\varphi}/(1+\varphi)$ denote period welfare. At fixed shocks, household optimality and cost minimization imply

$$du_t = \nu_t P_{H,t} \left[(1 - mc_t \Delta_t^p) dC_{H,t} - mc_t Y_t \, d\Delta_t^p \right] - \lambda_t \, dNX_t \tag{57}$$

The domestic wedge is now $1 - mc_t \Delta_t^p$: delivering an extra unit of consumption uses more inputs when prices differ across firms. The next term values the resource cost of changing that price dispersion. Combining the trade term with the external budgets recovers the financial wedge $\lambda_t - \beta r \lambda_{t+1} = \beta r \lambda_{t+1} \psi_t$, future foreign payments and inherited valuation. Unlike the finite economy's terminal settlement, the infinite-horizon economy carries bonds and reserves forward under the specified policy rule; their future consequences remain in welfare. Appendix F derives the complete decomposition, following the implementability-and-wedges approach of Farhi and Werning (2016).

A feasible intervention is useful when its combined marginal effect is positive. Holding the optimized monetary-only rates fixed, a small first-quarter reserve sale raises welfare for oil-only and combined shocks at domestic ownership shares zero, one half and one. In the infinite-horizon economy with

Calvo pricing, holding rates fixed still permits domestic spending and price dispersion to adjust, so those terms remain in the test. This derivative explains the additional policy margin; the reported welfare gains compare the fully optimized plans.

7 Calibration

7.1 Energy exposure and opening portfolios

Table 1 gives the Indian moments used to set expenditure shares, opening balance sheets, shock persistence and the scale of intervention. Financial quantities in the model are in foreign-good units. For comparison with Indian data, stocks and flows are reported as ratios to reference quarterly gross domestic product (GDP), denoted Y^v for this conversion. It is fixed at $Y^v = \bar{Y} - \bar{P}^O \bar{X}_E = 0.96734$ with reference gross output $\bar{Y} = 1$. Gross output includes imported production oil; domestic value added excludes it.

Table 1: Indian calibration targets and model counterparts

Target (unit)	Data	Model
Expenditure and production, 2021–22		
Direct imports / private consumption (%)	5.010	5.010
Direct imported petroleum / private consumption (%)	0.716	0.716
Imported production energy / gross output (%)	3.266	3.266
Government and external portfolios		
Dated government securities / annual GDP (%)	34.10	34.10
Foreign-held dated debt / quarterly GDP (%)	3.137	2.745
Official reserves / quarterly GDP (%)	73.31	73.31
Debt-type net foreign assets / quarterly GDP (%)	20.10	20.10
Government bond duration (years)	7.4	7.4
Quarterly persistence and policy scale		
Real oil price, autoregressive coefficient	0.854	0.85
Portfolio-debt liability ratio, autoregressive coefficient	0.838	0.84
95th percentile, absolute spot operations / opening reserves (%)	6.51	6.51

The model column gives the equal-ownership reference. The foreign-held debt counterpart is $[\bar{F} + (1-s)\bar{D}]/Y^v$, not total intermediary holdings. Bond ownership is measured in 2023–24, duration in September 2024 and external stocks in 2025Q4. Debt-liability persistence proxies for direct investors' bond demand. The last row sets the maximum reserve sale in a quarter. Sources and constructions are given below.

The 2021–22 supply-and-use accounts distinguish oil purchased directly by households from fuel imported by domestic producers (Ministry of Statistics and Programme Implementation, 2025; Petroleum Planning and Analysis Cell, 2026). Direct imports account for 5.010% of private consumption; direct petroleum alone accounts for 0.716%. Their ratio gives the within-import energy share $\omega = 0.14289$. Production oil accounts for 3.266% of reference gross output. With production elasticity $\nu_E = 0.40$, matching this cost share and reference output gives $\mu = 0.00019931$ and $A = 1.02263$ in (8). The CES weight is a technology coefficient, not the oil expenditure share. Appendix H gives the exact mapping. The elasticity is close to the benchmark in Bodenstein et al. (2011); it governs how easily producers substitute labor for fuel. Government securities outstanding amount to 34.1% of annual

gross domestic product (GDP), of which foreign portfolio investors hold 2.3% (Department of Economic Affairs, 2025). Their holdings therefore represent $4(0.341)(0.023) = 0.031372$ of quarterly output. I assign three quarters of this scale to direct foreign investors and one quarter to intermediaries. At equal domestic and foreign ownership, foreign-held domestic bonds are 2.745% of quarterly output because half of the intermediary position is domestically owned. This financial calibration uses the observed stock to set the total trading scale; it does not exactly match the foreign-owned stock after the ownership split. Ownership comparisons keep $\bar{F}$, $\bar{D}$, total intermediary capacity and initial national net foreign assets fixed. Changing s then changes both the foreign share of intermediary earnings and the part of inherited domestic debt held abroad. The bond's coupon decay $\delta_B = 0.96980$ gives a coupon-weighted duration of $1/[4(1-\delta_B/r)] = 7.4$ years, using the index's reported effective duration as its target (LSEG / FTSE Russell, 2025). The central bank initially holds 12.3% of the government stock. I take reserves and net foreign assets from the Reserve Bank of India's (RBI) international investment position, excluding equity and direct investment from the net position (Reserve Bank of India, 2026b). The broader foreign custody series includes instruments outside the modeled government debt (National Securities Depository Limited, 2025).

7.2 The shocks and policy instruments

I report an oil-only MIT shock and a combined oil and investor-mandate shock. The oil price initially rises ten percent. In the combined experiment, direct investors' foreign-currency bond target also falls by one percent of reference quarterly output. The two paths are

$$\log(P_t^O/\bar{P}^O) = 0.85^t \log(1.10), \qquad F_t - \bar{F} = -0.01Y^v(0.84)^t, \qquad t = 0, 1, \ldots$$

In the oil-only experiment $F_t = \bar{F}$. Both innovations occur in the first quarter, and subsequent innovations are zero. The realized paths are known when policy responds; their persistence does not introduce another surprise. Quarterly oil persistence comes from log real Brent (US consumer prices as deflator), 2013Q2–2025Q4 (U.S. Energy Information Administration, 2026; U.S. Bureau of Labor Statistics, 2026). Portfolio-debt liabilities relative to quarterly GDP give the investor persistence over 2018Q3–2025Q4. The shock sizes are about 0.69 and 2.1 historical residual standard deviations. Across policies I hold the target path fixed and calculate net foreign bond sales at the prices at which the trades occur.

In the Indian episodes, sales extended to both bonds and equities (Figure 2). The Indian basket rose from about \$85 per barrel in January 2022 to \$113 in March, when foreign investors were net sellers in both markets; they also sold both assets during the March 2026 oil-price increase. The model isolates the financing of bond sales.

Table 2 separates preference and trading parameters from the matched expenditure and portfolio moments. Equal ownership follows Cavallino (2019). The marginal-fee slope is $\gamma = 0.001/Y^v = 0.00103376$ when positions are in foreign-good units. Capacity exceeds the normal position by 25% of $\bar{D}$, or 0.196% of quarterly GDP. These specify the intermediary demand schedule and available financing capacity. The quarterly flow ceiling is 6.51% of opening reserves, the 95th percentile of absolute quarterly net spot operations over 2013Q2–2025Q4 (Reserve Bank of India, 2026a). The reserve floor protects 4.5 of the initial 7.72 months of import cover (International Monetary Fund, 2025). As

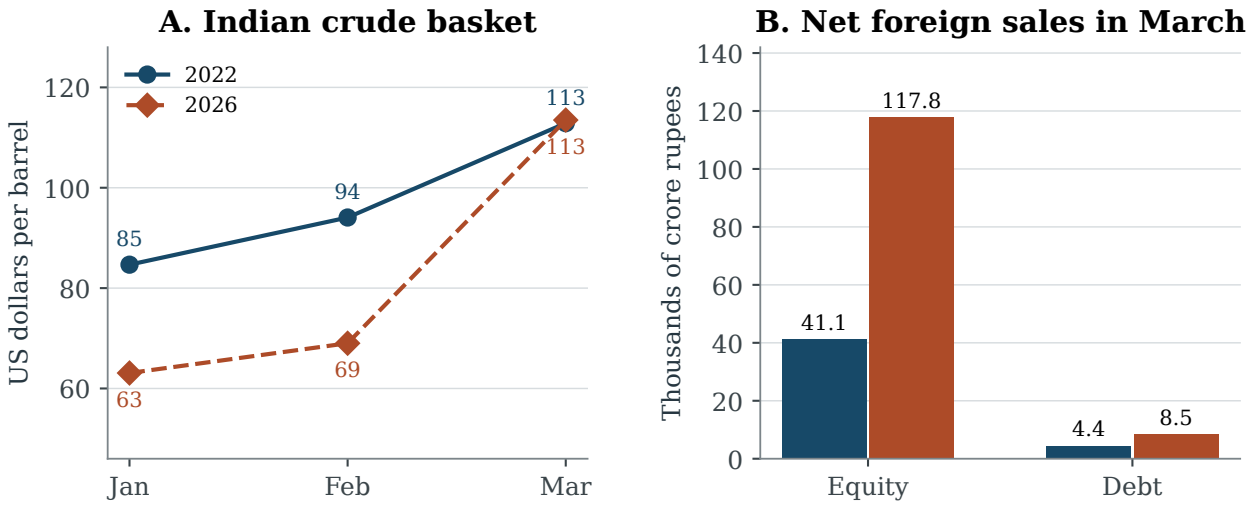


Figure 2: Simultaneous oil price increases and foreign sales of Indian securities

Panel A: monthly published Indian basket. Its composition changed in March 2026; the reported levels span that change. Panel B: March net foreign portfolio sales; one thousand crore equals ten billion rupees. Debt includes the voluntary retention route. Sources: Petroleum Planning and Analysis Cell (2026), Securities and Exchange Board of India (2022) and Securities and Exchange Board of India (2026).

Table 2: Preference, price-setting and financial parameters

Object	Value	Role
Discount factor; curvature of utility	0.9963; 1	Quarterly patience; log consumption
Inverse Frisch elasticity	1	Marginal cost of hours
Domestic–import; within-import elasticities	1; 1	Household substitution
Labor–energy substitution	0.40	Production input response
Price nonreset probability; variety elasticity	0.75; 6	Four-quarter mean price spell; price markup
Marginal-fee slope; exposure per unit of capital	0.001034; 5	Interior pricing; units of capacity
Available capacity; domestic ownership share	25%; 50%	Benchmark for comparisons
Maximum quarterly log rate change	0.005	Operating limit
Policy horizon	24 quarters	Rate and reserve decisions

ratios to reference GDP, the opening reserve stock is $R_{-1}/Y^{v} = 0.73310$, the floor $\underline{R}/Y^{v} = 0.42749$, and the intervention limit $\bar{Z}/Y^{v} = 0.04775$. Wages remain flexible; Calvo pricing and the preference benchmarks are standard in the commodity-policy literature (Drechsel et al., 2026).

7.3 Policy comparisons and realized net sales

The policy comparisons use the same shocks, inherited portfolios and operational bounds:

- **Taylor rule:** the policy rate follows the rule below and $Z_t = 0$ throughout.
- **Monetary policy alone:** choose the admissible rate path to maximize welfare, with $Z_t = 0$ throughout.
- **Joint policy:** choose both the rate path and reserve trades to maximize welfare.

- **Delayed joint policy:** impose $Z_0 = 0$ on joint policy; rates can still respond in the first quarter.

The Taylor rule is

$$\log(I_t/r) = 0.75\log(I_{t-1}/r) + 0.25[1.5\log(P_t/P_{t-1}) + 0.0625\log(Y_t/\bar{Y})] \tag{58}$$

The first three policies establish what optimized monetary policy and the addition of FXI achieve. The delayed joint plan isolates the value of access to reserves in the shock quarter: all later instruments remain available, and the first-quarter rate is still optimized. Households anticipate the announced plan in each comparison.

At current transaction prices, nonresidents' net sale of bonds is

$$\text{Sale}_t = \frac{Q_t^B}{S_t Y^v}[\delta_B B_{t-1}^{ext} - B_t^{ext}], \qquad B_t^{ext} = B_t^F + (1-s)B_t^D \tag{59}$$

The measure excludes coupons and cancels trades between two nonresidents. The target F_t fixes direct investors' holdings at the end of the period, valued in foreign currency. I report welfare in permanent consumption-equivalent basis points (CE bp): $10^4\{\exp[(1-\beta)\Delta\mathcal{W}] - 1\}$, where $\Delta\mathcal{W}$ is the gain in discounted welfare. With log aggregate consumption this is the proportional consumption increase in every quarter that makes the lower-welfare allocation equally desirable, holding its hours fixed. It is distinct from either an impact consumption percentage or a flow measured relative to GDP.

8 Results

8.1 Oil alone creates financial pressure

Under the Taylor rule, the oil-only shock lowers first-quarter consumption by 0.342% and depreciates the currency by 1.359%. Direct foreign investors retain their target, but the higher import bill still requires financing after spending and inherited debt payments adjust. Table 3 separates these contributions. The fall in the foreign-currency payment on outstanding bonds covers about half of the initial trade shortfall; intermediaries finance the remainder and demand a positive premium. This is the quantitative counterpart of Proposition 2 under a specified monetary response.

For unchanged opening portfolios, the first-quarter stock-flow identity is

$$D_0 - \bar{D} = -(F_0 - \bar{F}) - (NX_0 - \overline{NX}) + (R_0^B - r)\bar{V} - Z_0, \tag{60}$$

where $\bar{V} = \bar{F} + (1-s)\bar{D}$. The four terms are the change in direct investors' target, the trade shortfall, the change in inherited foreign payments, and reserve sales. Their sum determines the change in intermediary financing. This identity accounts for all foreign bond transactions, including purchases needed to maintain an unchanged mandate after valuation losses.

Optimizing monetary policy alone changes spending and prices but leaves a similar initial financing requirement. The central bank can lower the premium by inducing a larger external adjustment, yet that adjustment also changes household consumption and employment. At the monetary-only optimum, further changes in rates must be judged against those effects throughout the path.

Table 3: Oil only: financing the first-quarter trade shortfall

	Taylor	Monetary alone	Joint
Cut in investor target	0.0000	0.0000	0.0000
Deterioration in net exports	0.1007	0.1005	0.1906
Change in inherited foreign payment	-0.0503	-0.0520	-0.0142
Less: reserve sales	0.0000	0.0000	0.1923
Increase in intermediary position	0.0504	0.0485	-0.0158

Percent of reference quarterly GDP; equal domestic and foreign intermediary ownership. The first three rows minus reserve sales equal the increase in intermediary holdings, before rounding. A negative inherited-payment entry is a smaller payment to existing foreign claim holders.

Joint policy sells reserves worth 0.192% of quarterly GDP in the first quarter. Consumption falls by 0.272%, compared with 0.380% under optimized monetary policy alone. The trade deficit is larger under joint policy because households maintain more of their purchases; the additional bill is financed with reserves rather than a larger intermediary position. Its premium is close to zero and slightly negative, so intervention is not being chosen to hit a zero-premium constraint. The initial exchange rate is also close to its reference level, with an appreciation of 0.094% (Table 4).

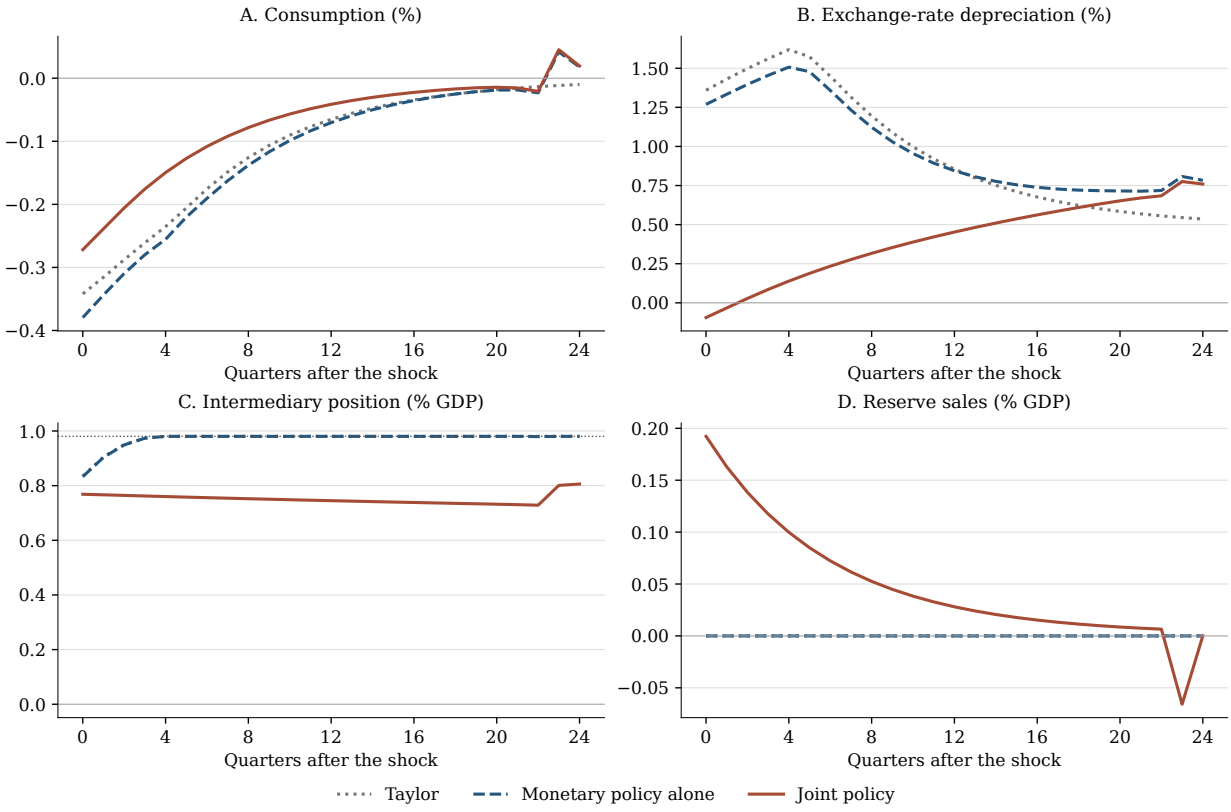


Figure 3: Oil only: consumption and the financing response

A ten percent initial oil-price rise with persistence 0.85 and no investor-mandate innovation; $s = 0.5$. Consumption and exchange rates are changes from the stationary reference. Positive depreciation means a weaker domestic currency. The dotted horizontal line in panel C is the intermediary position limit. The central bank's optimized controls end after quarter 23; the Taylor continuation begins in quarter 24.

Table 4: Oil only: first-quarter allocations and transactions

Policy	Consumption	Depreciation	Premium	Reserves	Net sales
Taylor	-0.342	1.359	0.0050	0.000	-0.157
Monetary policy alone	-0.380	1.269	0.0048	0.000	-0.158
Joint policy	-0.272	-0.094	-0.0016	0.192	-0.089

Consumption and depreciation are percent changes. The premium is in quarterly return basis points; reserve sales and net nonresident bond sales are percent of reference quarterly GDP. Negative net sales are purchases. No portfolio-demand innovation occurs in this experiment.

Why does a higher import bill coexist with higher welfare? Oil itself remains costly under either policy. Monetary policy alone must weigh the domestic wedge against the financial wedge as it changes spending and prices. Reserves allow an additional financing adjustment, so a smaller initial consumption cut can be worthwhile even when it leaves a larger import bill. The welfare test includes domestic production, price dispersion, foreign payments and the reserve position carried forward, as in (57) and Appendix F. A small reserve sale at the optimized monetary-only plan has a positive net derivative. The full reoptimized joint plan improves lifetime welfare by 0.002747 CE bp.

The first-quarter consumption fall is not a sufficient welfare statistic. Taylor policy has a smaller initial consumption contraction than optimized monetary policy alone, although the latter gives higher lifetime welfare. The planner also values later consumption and the hours required to produce domestic goods. Table 6 reports those full-path comparisons rather than ranking policies from a single response.

8.2 The policy mix under the combined shock

When the oil increase coincides with the reduction in foreign investors' target, intermediaries reach their position limit in the first quarter. Under optimized monetary policy alone, a higher trade surplus provides 0.499% of quarterly GDP, the smaller payment on inherited foreign bonds provides 0.305%, and intermediaries absorb another 0.196%. These adjustments together accommodate the one-percent cut in the mandate. The country pays for the shift in foreign demand partly through a larger contraction and depreciation.

Joint policy supplies reserves worth 0.256% of quarterly GDP on impact. Consumption falls by 0.763% rather than 0.996%, and depreciation falls from 11.613% to 8.381%. The weaker contraction permits a smaller improvement in net exports, while the smaller depreciation gives up part of the saving on inherited foreign debt. Both effects are included in the planner's choice. The intervention finances a different real adjustment; it does not remove the income loss from more expensive oil.

Table 5: Combined shock: first-quarter policy and allocations

Policy	Consumption	Depreciation	Premium	Reserves	Net sales
Taylor	-0.905	11.871	901.2479	0.000	0.523
Monetary policy alone	-0.996	11.613	907.1973	0.000	0.524
Joint policy	-0.763	8.381	867.5100	0.256	0.597
Delayed joint	-0.998	11.608	1135.6419	0.000	0.524

Same units as Table 4; $s = 0.5$. The shock is a ten percent oil-price rise and a one-percent-of-GDP cut in the investor mandate. Delayed joint policy cannot sell reserves in the first quarter but can choose the interest rate immediately.

The policy mix also moderates inflation. Initial annualized consumer price inflation is 2.08 percentage points above the reference under joint policy, compared with 2.66 under monetary policy alone. The optimized interest-rate increase is smaller, and households face a lower initial rise in the domestic price of imports (Figure 4). Intermediaries still earn a substantial scarcity premium at the limit. Additional intervention would alter reserve income, future financing and the value of inherited bonds, so the optimal joint policy need not eliminate that premium.

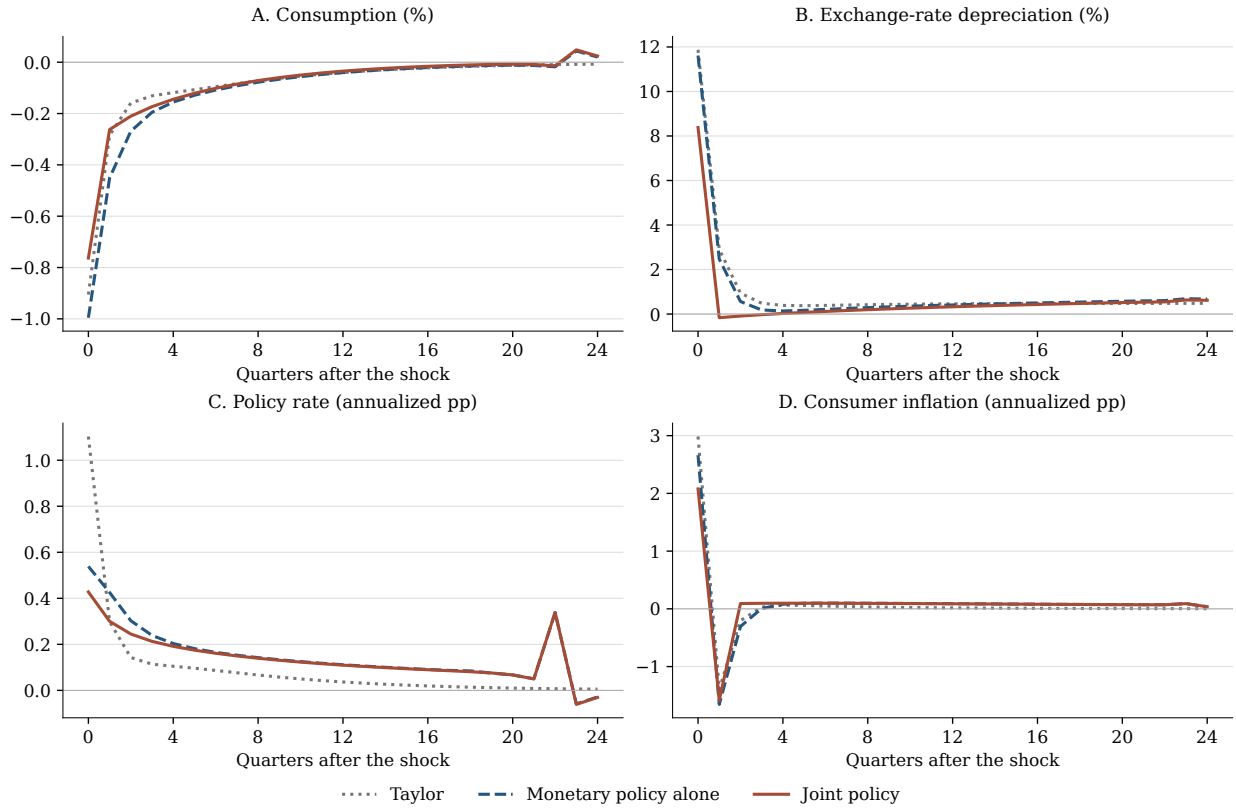


Figure 4: Combined shock: consumption, the exchange rate and monetary policy

Identical oil and investor-mandate paths under all three policies; $s = 0.5$. Rate and inflation changes are annualized percentage points. Changes in slopes can reflect switches between binding and slack intermediary capacity; the policy window and continuation rule are common across the optimized plans.

Realized nonresident sales differ across policies even though the mandate shock is identical. They are 0.524% of GDP under monetary policy alone and 0.597% under joint policy. A given foreign-currency target corresponds to different face holdings when bond prices and the exchange rate change. Foreign-owned intermediaries can also buy bonds sold by direct investors. The transaction measure in (59) includes both groups and excludes revaluation and coupons; the reported difference is an equilibrium response to the same shock, not a different imposed outflow.

The joint gain over optimized monetary policy is 0.003111 CE bp in the combined experiment. These permanent consumption equivalents average a temporary policy improvement over the household's lifetime. The impact responses in Table 5 describe where the improvement occurs, while the welfare measure includes the future cost of financing it.

Table 6: Welfare relative to Taylor policy for the same shock

Shock	Taylor	Monetary alone	Joint	Delayed joint
Oil only	0.000000	0.000681	0.003427	0.003427
Combined	0.000000	0.002289	0.005401	0.003667

Permanent consumption-equivalent basis points, with $s = 0.5$. Every entry compares policies with the same shock and inherited portfolios. The oil-only immediate and delayed gains agree at the displayed precision.

8.3 When waiting is costly

The timing comparison keeps the shock and the entire monetary-policy menu unchanged. The central bank can announce a plan immediately, but delayed joint policy must wait until the second quarter to trade reserves. Thus households anticipate future intervention; they cannot use next quarter's reserves to settle this quarter's trade.

In the oil-only experiment, the optimized monetary-only plan reaches the intermediary limit later in the adjustment. Immediate and delayed joint policy both avoid binding capacity at the benchmark ownership share. Private financing can accommodate the first oil bill until the reserve operation occurs, so the two consumption and welfare paths are almost identical. The cost of a one-quarter delay is below 0.000001 CE bp. A valuable instrument need not be valuable in every quarter: the benefit of access to FXI over the whole path can be positive even when excluding its first use changes welfare very little.

The combined shock is different because the foreign target falls sharply in the same quarter as the first oil bill arrives. Intermediaries reach their upper limit immediately under both immediate and delayed joint policy. Without a current reserve sale, the initial private adjustment must clear the market at that fixed position. The first-quarter premium rises from about 868 quarterly basis points under immediate joint policy to 1,136 with delay. The return is promised on the first-quarter position and paid in the following quarter; later intervention cannot retroactively change that contract.

Waiting costs 0.001734 CE bp, or 55.7% of the gain from adding FXI to optimized monetary policy. Consumption initially falls by 0.998% rather than 0.763%, and intermediary capacity binds for two quarters rather than one. This loss combines the different initial real adjustment, payments promised on the initial position and the later paths needed to restore equilibrium. It is not simply the current premium multiplied by a fixed amount of borrowing.

8.3.1 Who receives the premium?

Domestic ownership changes the national incidence of intermediary returns. A higher return on a domestically owned position accrues to domestic households, while the foreign-owned share is paid abroad. When delay raises subsequent intermediary earnings, a larger domestic share retains more of those earnings at home. Ownership also changes which residents bear the first-period revaluation of existing positions, so the welfare comparison reoptimizes both plans rather than mechanically subtracting a foreign-payment share.

The cost of waiting under the combined shock falls from 0.003273 CE bp at full foreign ownership to 0.000603 at full domestic ownership (Table 7). It does not vanish at $s = 1$. Intermediaries still face a private marginal fee and a position limit, fund themselves in the foreign safe market, and coexist with direct foreign investors. Rebating the fee removes a national resource loss; it does not make

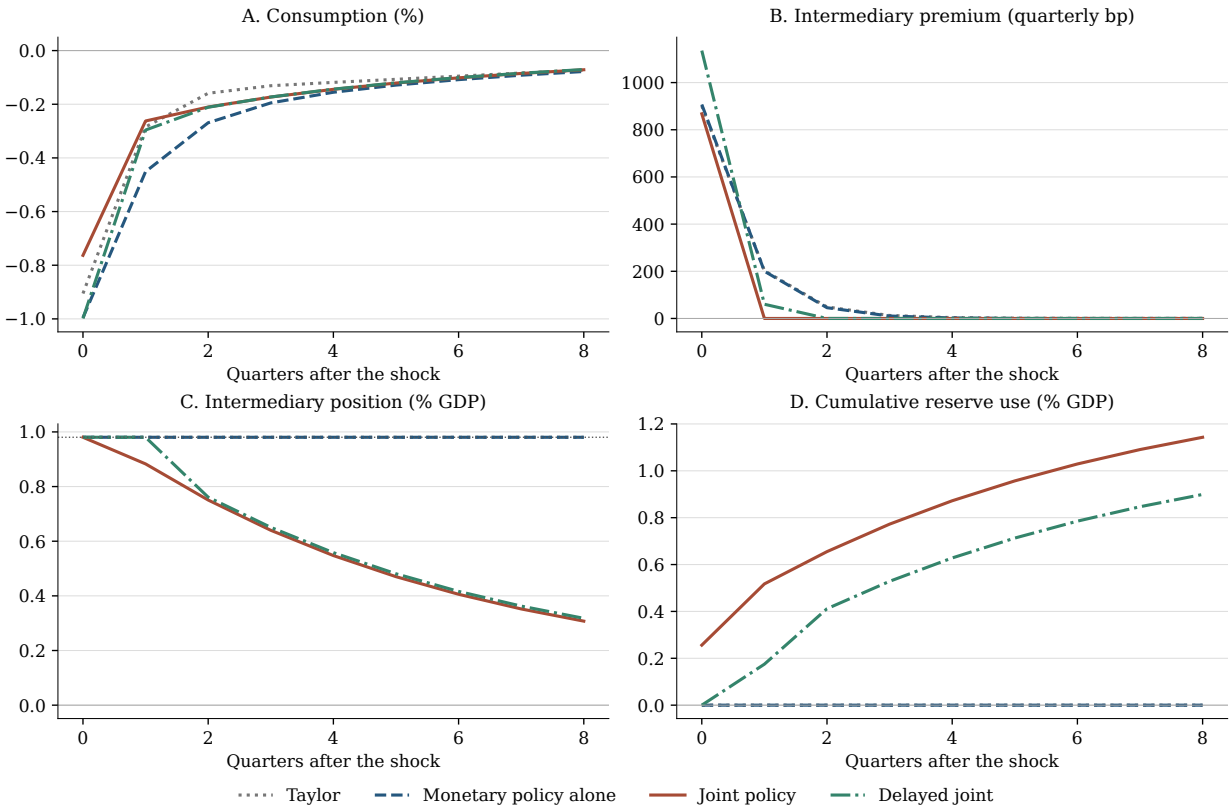


Figure 5: Combined shock: the initial trade and the cost of waiting

$s = 0.5$. Delayed joint policy allows the same immediate interest-rate choice as joint policy and imposes only $Z_0 = 0$. The dotted horizontal line in panel C is the common position limit. Cumulative reserve use is the opening stock less the current stock, divided by reference quarterly GDP.

Table 7: Intermediary ownership and the value of intervention

Domestic share	Oil: FXI gain	Combined: FXI gain	Combined: delay cost
0%	0.003244	0.004818	0.003273
25%	0.002998	0.003920	0.002465
50%	0.002747	0.003111	0.001734
75%	0.002490	0.002521	0.001105
100%	0.002232	0.003405	0.000603

Permanent CE bp. The FXI gain compares joint policy with optimized monetary policy alone. Delay cost compares immediate with delayed joint policy. Opening direct-investor and intermediary positions, total capacity and national net foreign assets are fixed as ownership changes.

unrestricted foreign financing available to households. With sticky domestic prices, the change in the initial financing opportunity can still alter consumption, employment and future payments in a costly way.

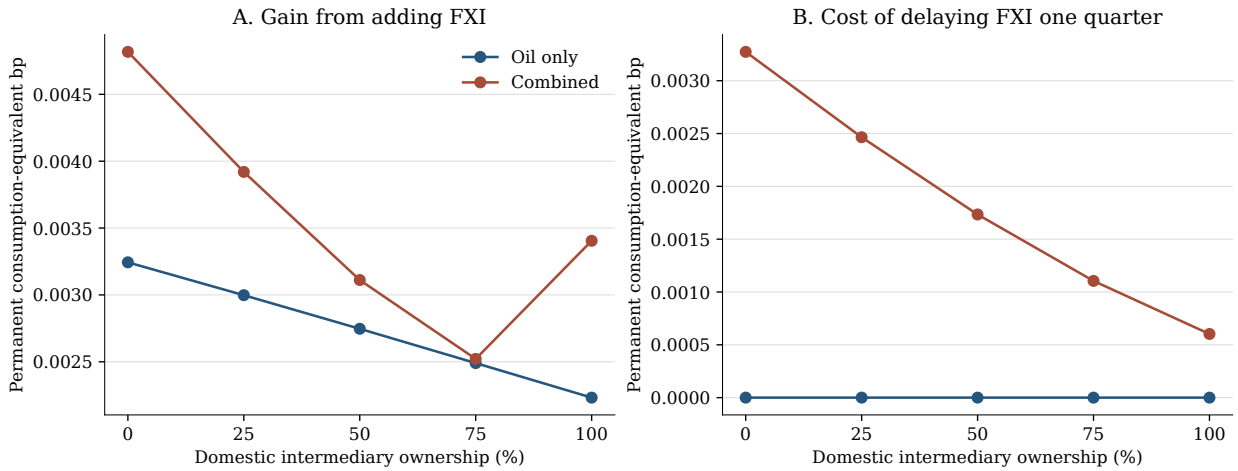


Figure 6: Ownership changes the value of early intervention

All policy plans are reoptimized at each ownership share. The combined-shock cost of delay decreases over the grid. The total gain from adding FXI need not be monotone: ownership changes inherited valuation and the full intertemporal allocation as well as the foreign incidence of intermediary earnings.

The oil-only joint gain is also positive with fully domestic intermediaries. The financial wedge and nominal rigidities remain in that economy, and direct foreign investors still receive bond payments. The independent feasible-intervention check in Appendix F confirms a positive net welfare derivative in this case without charging households for a domestic fee that is rebated. The mechanism therefore concerns constrained financing and the allocation it supports, rather than a resource cost assigned to domestic intermediary profits.

9 Conclusion

An oil importer must finance a more expensive input even after households and firms adjust their purchases. The two-period economy shows how a higher oil price can raise intermediary positions and the premium, without a foreign-investor withdrawal. Import spending and inherited debt values adjust, while repayment links the initial oil bill to household choices in the following period.

This financing channel gives the central bank a reason to use reserves alongside monetary policy. Monetary policy affects demand, production and the value of outstanding domestic-currency debt. A reserve sale changes intermediary financing and returns on foreign-held claims. The welfare decomposition identifies when this additional instrument improves on optimized monetary policy, accounting for inherited valuation and repayment.

The infinite-horizon economy adds staggered domestic prices and persistent responses to the initial shock. In the Indian calibration, joint policy cushions consumption after an oil shock and moderates contraction and depreciation when portfolio withdrawals accompany it. Households can sustain a

larger import bill because the value of their consumption exceeds the additional financing, production and reserve costs.

Intervention timing matters when intermediary capacity binds immediately. Waiting until the next quarter changes initial spending and promised bond returns, sacrificing more than half of the joint-policy gain in the combined experiment. The loss falls with domestic ownership as more earnings return to households. The position limit and financing wedge remain, leaving a positive cost of delay.

Future work will add foreign equity holdings and study the Indian episodes, separating bond-market financing from sales of claims on firm profits. Recurring shocks would introduce uncertainty about future financing needs. Comparing monetary regimes would show how an exchange-rate commitment changes the optimal mix.

References

Adrian, T., Azour, J., Chalk, N., Gourinchas, P.-O., Kammer, A., Selassie, A. A., Srinivasan, K., and Valdés, R. (2026). How the war in the middle east is affecting energy, trade, and finance. IMF Blog.

Aktuğ, E. and Rezghi, A. (2026). Optimal exchange rate policy with oil shocks. *IMF Economic Review*.

Basu, S. S., Boz, E., Gopinath, G., Roch, F., and Unsal, D. F. (2025). Integrated monetary and financial policies for small open economies. *Econometrica*, 93(6):2201–2234.

Bergholt, D., Larsen, V. H., and Seneca, M. (2019). Business cycles in an oil economy. *Journal of International Money and Finance*, 96:283–303.

Bianchi, J., Bigio, S., and Engel, C. (2021). Scrambling for dollars: International liquidity, banks and exchange rates. Working Paper 29457, National Bureau of Economic Research.

Bodenstein, M., Erceg, C. J., and Guerrieri, L. (2008). Optimal monetary policy with distinct core and headline inflation rates. *Journal of Monetary Economics*, 55:S18–S33.

Bodenstein, M., Erceg, C. J., and Guerrieri, L. (2011). Oil shocks and external adjustment. *Journal of International Economics*, 83(2):168–184.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Catão, L. and Chang, R. (2013). Monetary rules for commodity traders. *IMF Economic Review*, 61:52–91.

Cavallino, P. (2019). Capital flows and foreign exchange intervention. *American Economic Journal: Macroeconomics*, 11(2):127–170.

Central Depository Services (India) Limited (2026). Foreign portfolio investor investment data. CDSL statistical releases.

Chan, J., Diz, S., and Kanngiesser, D. (2024). Energy prices and household heterogeneity: Monetary policy in a Gas-TANK. *Journal of Monetary Economics*, 147:103620.

Chang, C., Liu, Z., and Spiegel, M. M. (2015). Capital controls and optimal Chinese monetary policy. *Journal of Monetary Economics*, 74:1–15.

Department of Economic Affairs (2025). Status paper on government debt for 2023–24. Government of India, Ministry of Finance.

Drechsel, T., McLeay, M., and Tenreyro, S. (2019). Monetary policy for commodity booms and busts. In *Challenges for Monetary Policy*. Federal Reserve Bank of Kansas City.

Drechsel, T., McLeay, M., Tenreyro, S., and Turri, E. D. (2026). Optimal monetary and exchange rate frameworks for commodity-exposed economies. *Journal of International Economics*, 164:104330.

Egorov, K. and Mukhin, D. (2023). Optimal policy under dollar pricing. *American Economic Review*, 113(7):1783–1824.

Faltermeier, J., Lama, R., and Medina, J. P. (2022). Foreign exchange intervention for commodity booms and busts. *European Economic Review*, 143:104018.

Fanelli, S. (2026). Monetary policy, capital controls, and international portfolios. CEMFI, April. Accepted, Review of Economic Studies.

Fanelli, S. and Straub, L. (2021). A theory of foreign exchange interventions. *Review of Economic Studies*, 88(6):2857–2885.

Farhi, E. and Werning, I. (2016). A theory of macroprudential policies in the presence of nominal rigidities. *Econometrica*, 84(5):1645–1704.

Gabaix, X. and Maggiori, M. (2015). International liquidity and exchange rate dynamics. *Quarterly Journal of Economics*, 130(3):1369–1420.

Hevia, C. and Nicolini, J. P. (2013). Optimal devaluations. *IMF Economic Review*, 61(1):22–51.

Hevia, C. and Nicolini, J. P. (2015). Monetary policy and Dutch disease: The case of price and wage rigidity. In Caputo, R. and Chang, R., editors, *Commodity Prices and Macroeconomic Policy*, volume 22 of *Central Banking, Analysis, and Economic Policies*, pages 51–89. Central Bank of Chile.

International Monetary Fund (2025). Assessing reserve adequacy—ara metric. International Monetary Fund Data.

Iovino, L. and Sergeyev, D. (2023). Central bank balance sheet policies without rational expectations. *Review of Economic Studies*, 90(6):3119–3152.

Itskhoki, O. and Mukhin, D. (2023). Optimal exchange rate policy. Working Paper 31933, National Bureau of Economic Research.

Jin, H. and Xiong, C. (2021). Fiscal stress and monetary policy stance in oil-exporting countries. *Journal of International Money and Finance*, 111:102302.

Klump, R., McAdam, P., and Willman, A. (2011). The normalized CES production function: Theory and empirics. Working Paper 1294, European Central Bank.

LSEG / FTSE Russell (2025). A resilient indian fixed income market. Government-bond index characteristics as of April 2025.

Maggiori, M. (2022). International macroeconomics with imperfect financial markets. In Gopinath, G., Helpman, E., and Rogoff, K., editors, *Handbook of International Economics*, volume 6, pages 199–236. Elsevier.

Ministry of Statistics and Programme Implementation (2025). Supply and use tables, 2020–21 and 2021–22. Government of India.

Mukhin, D. (2022). An equilibrium model of the international price system. *American Economic Review*, 112(2):650–688.

National Securities Depository Limited (2025). Foreign portfolio investors: Assets under custody. Monthly report, December 2025.

Omotosho, B. S. (2022). Oil price shocks and monetary policy in resource-rich economies: Does capital matter? *Journal of Economic Dynamics and Control*, 143:104479.

Petroleum Planning and Analysis Cell (2026). India's petroleum statistics. Ministry of Petroleum and Natural Gas, Government of India.

Reserve Bank of India (2026a). Handbook of statistics on the indian economy. Database on Indian Economy.

Reserve Bank of India (2026b). India's balance of payments and international investment position. Database on Indian Economy.

Securities and Exchange Board of India (2022). SEBI bulletin, april 2022.

Securities and Exchange Board of India (2026). SEBI bulletin, april 2026.

U.S. Bureau of Labor Statistics (2026). Consumer price index for all urban consumers: All items in u.s. city average. FRED series CPIAUCSL, Federal Reserve Bank of St. Louis.

U.S. Energy Information Administration (2026). Europe brent spot price fob. FRED series DCOILBRENTEU, Federal Reserve Bank of St. Louis.

A Two-period household, firm and financial accounts

A.1 Household optimization and public accounts

Preferences are exactly (1). The monetary representation selects aggregate domestic expenditure subject to private optimality; it adds no term to utility and no restriction to an individual household's consumption choices.

Bond clearing and the reserve trade are (22)–(23) and (15). The surviving bond pays L in the second period. The short domestic claim has price $Q_0^S = 1/I_0$, payoff one, and zero net supply.

Write T_t for the net public transfer to households. The consolidated public budgets in the finite economy are

$$T_0 + Q_0^B(B_0^{CB} - \delta_B B_{-1}^{CB}) + S_0(R_0 + \bar{G}) + B_{-1}^G = B_{-1}^{CB} + S_0 r(R_{-1} + \bar{G}), \tag{61}$$

$$T_1 + \delta_B L B_{-1}^G = L B_0^{CB} + S_1 r(R_0 + \bar{G}) \tag{62}$$

Here the predetermined nonreserve position is included in the consolidated public account and is settled with all remaining assets. It carries no household portfolio choice. Placing its predetermined income in a household transfer account gives the same allocation and external budget.

Domestic intermediary distributions, including the fee rebates, are (20). Any predetermined capital return can be consolidated with the inherited foreign position and transfers. The fee and its rebate enter with opposite signs, leaving no additional loss of goods.

The household chooses $(C_{H,t}, C_{M,t}, C_{E,t}, N_t)_{t=0,1}$ and (B_0^S, H_0) subject to (2)–(3). Let $\mathcal{H}_t$ denote income minus expenditure in those budgets. The Lagrangian is

$$\mathfrak{L}^H = \sum_{t=0}^{1} \beta^t \big[(1-\alpha)\log C_{H,t} + \alpha(1-\omega)\log C_{M,t} + \alpha\omega \log C_{E,t} - \chi N_t + \nu_t \mathcal{H}_t\big] + \zeta^H H_0,$$

where $\zeta^H \geq 0$ enforces $H_0 \geq 0$. Its goods and labor conditions are

$$\frac{1-\alpha}{C_{H,t}} = \nu_t, \qquad \frac{\alpha(1-\omega)}{C_{M,t}} = \nu_t S_t, \qquad \frac{\alpha\omega}{C_{E,t}} = \nu_t S_t P_t^O,$$
$$\chi = \nu_t W_t \tag{63}$$

The two asset conditions are

$$\nu_0 Q_0^S = \beta \nu_1, \qquad \nu_0 Q_0^B = \beta L \nu_1 + \zeta^H, \qquad \zeta^H H_0 = 0 \tag{64}$$

At positive long-bond holdings these equations give $I_0 Q_0^B = L$. Dividing the goods conditions gives (4) and (5); dividing the nominal multipliers across periods gives $C_{H,1} = \beta I_0 C_{H,0}$.

Concavity of utility and linear budgets make these private conditions sufficient at positive consumption. Linear disutility of work can make labor supply set-valued at the market wage; the stated employment is supported by labor-market clearing. The result does not require strictly convex labor disutility in the finite economy.

A.2 Input demands and the trade derivatives

Cost minimization has Lagrangian

$$\begin{aligned}\mathfrak{L}^F = {} & W_t N_t + S_t P_t^O X_{E,t} \\ & + mc_t \left[C_{H,t} + \bar{X} - A \left[(1-\mu) {N_t}^{(\nu_E - 1)/\nu_E} + \mu {X_{E,t}}^{(\nu_E - 1)/\nu_E} \right]^{\nu_E/(\nu_E - 1)} \right]\end{aligned}$$

Positive factor prices and output make the production constraint bind. The two factor conditions and constant returns give

$$W_t = mc_t \frac{\partial Y_t}{\partial N_t}, \qquad S_t P_t^O = mc_t \frac{\partial Y_t}{\partial X_{E,t}},$$

$$mc_t = \frac{1}{A} \left[(1-\mu)^{\nu_E} W_t^{1-\nu_E} + \mu^{\nu_E} (S_t P_t^O)^{1-\nu_E} \right]^{1/(1-\nu_E)}, \tag{65}$$

$$N_t = Y_t A^{\nu_E - 1} (1-\mu)^{\nu_E} \left(\frac{mc_t}{W_t} \right)^{\nu_E}, \qquad X_{E,t} = Y_t A^{\nu_E - 1} \mu^{\nu_E} \left(\frac{mc_t}{S_t P_t^O} \right)^{\nu_E} \tag{66}$$

Household optimality gives $W_t / S_t = \chi z_t / \alpha$. Substitution gives the production-oil bill

$$P_t^O X_{E,t} = \frac{C_{H,t} + \bar{X}}{A} \mu^{\nu_E} (P_t^O)^{1-\nu_E} \left[(1-\mu)^{\nu_E} (\chi z_t / \alpha)^{1-\nu_E} + \mu^{\nu_E} (P_t^O)^{1-\nu_E} \right]^{\nu_E/(1-\nu_E)} \tag{67}$$

This is production oil; household oil remains $\omega z_t / P_t^O$. Continuous limits apply when $\nu_E = 1$.

For the proofs below, a subscript z or P denotes a partial derivative holding domestic spending fixed; P refers to the current oil price. Let the oil share of unit cost, used only to state these derivatives, be

$$\theta_{E,t} = \frac{\mu^{\nu_E} (P_t^O)^{1-\nu_E}}{(1-\mu)^{\nu_E} (\chi z_t / \alpha)^{1-\nu_E} + \mu^{\nu_E} (P_t^O)^{1-\nu_E}}$$

Differentiating (67) gives

$$-NX_{t,z} = 1 + \nu_E (1 - \theta_{E,t}) P_t^O X_{E,t} / z_t > 0, \tag{68}$$

$$-NX_{t,P} = X_{E,t} [1 - \nu_E (1 - \theta_{E,t})] > 0 \quad \text{if } 0 < \nu_E \leq 1 \tag{69}$$

The oil-price derivative holds total household foreign spending z_t fixed, so the household's more expensive oil purchase is offset by its intratemporal allocation. The direct increase in the trade bill in (69) comes from production oil. Household oil still enters utility, import prices and the monetary-policy response.

A.3 Aggregation of the financial accounts

Substitute profits, public transfers and bond clearing into the household budgets. The first-period aggregate resource equation becomes

$$\bar{X}^* - z_0 - P_0^O X_{E,0} = \left[A_0 - rA_{-1} + (R_0^B - r) V_{-1} \right]$$

The second-period equation is

$$\bar{X}^* - z_1 - P_1^O X_{E,1} = [-rA_0 + r\psi_0 V_0]$$

These are (26)–(27). In the second period, the terminal return L is paid on every surviving private and central-bank bond, and the public foreign position is settled. No outstanding asset is discarded.

B Implementability and the conditional equilibrium

B.1 Proof of Proposition 1

Necessity follows by combining the household first-order conditions, firms' cost minimization, intermediary optimality and the preceding budget aggregation. In particular,

$$I_0 \frac{S_0}{S_1} = \frac{C_{H,1}}{\beta C_{H,0}} \frac{C_{H,0}/z_0}{C_{H,1}/z_1} = \frac{z_1}{\beta z_0} = r(1+\psi_0),$$

which proves (28). This calculation uses domestic asset pricing only.

For sufficiency, take an allocation satisfying the proposition's restrictions. Construct

$$S_t = \frac{\alpha C_{H,t}}{(1-\alpha) z_t}, \qquad W_t = \frac{\chi C_{H,t}}{1-\alpha}$$

Select the candidate's domestic expenditure levels as the monetary settings. Recover $I_0 = C_{H,1}/(\beta C_{H,0})$ and $Q_0^B = L/I_0$. Foreign investor and intermediary face holdings are $S_0 F_0/Q_0^B$ and $S_0 D_0/Q_0^B$. Recover central-bank holdings from (15) and household holdings from (23); they are admissible by hypothesis. Set short-claim holdings to zero. Choose the public transfers from (61)–(62).

The two external budgets imply the household budgets after these substitutions. Goods demands, labor supply and saving satisfy the household conditions. Concavity supports household optimality. CES cost minimization supplies firms' optimal factor demands at their preset output price. Strict concavity of (13), together with its multiplier conditions, supplies intermediary optimality. Foreign mandates and all markets clear by construction. These steps establish implementation within the stated demand-control policy set.

The proposition conditions on positive household long-bond holdings. At a zero holding, (64) must retain its inequality multiplier; one cannot impose long-short return equality without that condition. The maintained analytical branch and every reported quantitative policy path have positive holdings.

B.2 Private equilibrium for a specified policy

With $(C_{H,0}, C_{H,1}, Z_0)$ fixed, domestic spending, I_0 and Q_0^B are fixed. For any proposed D_0, the first budget has derivative

$$NX_{0,z} - V_{-1} R_0^B / z_0 < 0$$

with respect to z_0. Hence it has at most one positive root, and

$$\frac{dz_0}{dD_0} = \frac{1}{-NX_{0,z} + V_{-1} R_0^B / z_0} > 0$$

Inside intermediary capacity, $z_1 = z_0[1 + \gamma(D_0 - \bar{D})]$. Differentiating the second-budget residual with respect to D_0 gives

$$-r[1 + (1-s)\psi_0] - r\gamma V_0 + NX_{1,z} \left[\frac{1+\psi_0}{-NX_{0,z} + V_{-1} R_0^B / z_0} + \gamma z_0 \right] < 0$$

At a position limit, D_0 is fixed and the residual's derivative with respect to ψ_0 is $NX_{1,z}z_0 - rV_0 < 0$. These signs require nonnegative foreign claims and positive gross returns. The branches meet at zero scarcity multipliers. When a root is bracketed on the admissible interval, these monotonicity properties select the private equilibrium; they do not assert existence for every proposed monetary setting.

C Oil and withdrawal transmission

C.1 Proof of Proposition 2

Hold monetary settings, the investor mandate and Z_0 fixed. The oil-price rise occurs only in the first period. Write a derivative subscript P for the total equilibrium response to P_0^O. The first budget and the stock identity give

$$D_{0,P} = (-NX_{0,z} + V_{-1}R_0^B/z_0)z_{0,P} - NX_{0,P} \tag{70}$$

Inside capacity, $\psi_{0,P} = \gamma D_{0,P}$. Differentiating the repayment budget and the household financial wedge gives

$$(-NX_{1,z})z_{1,P} + r[1 + (1-s)\psi_0 + \gamma V_0]D_{0,P} = 0, \tag{71}$$

$$z_{1,P} = (1+\psi_0)z_{0,P} + \gamma z_0 D_{0,P} \tag{72}$$

Both $-NX_{t,z}$ and $-NX_{0,P}$ are positive by (68)–(69). Also $1 + (1-s)\psi_0 > 0$ whenever $1 + \psi_0 > 0$. Eliminating $z_{1,P}$ and then $z_{0,P}$ yields

$$D_{0,P} = \frac{(-NX_{0,P})(-NX_{1,z})(1+\psi_0)}{(-NX_{1,z})(1+\psi_0) + (-NX_{0,z} + V_{-1}R_0^B/z_0)\{\gamma(-NX_{1,z})z_0 + r\gamma V_0 + r[1+(1-s)\psi_0]\}} > 0 \tag{73}$$

Every factor in the denominator is positive. Substituting this solution back into (70)–(72) gives $z_{0,P} < 0$ and $z_{1,P} < 0$, while $\psi_{0,P} = \gamma D_{0,P} > 0$. At a strictly binding upper position, $D_{0,P} = 0$. The first budget gives

$$z_{0,P} = \frac{NX_{0,P}}{-NX_{0,z} + V_{-1}R_0^B/z_0} < 0$$

Use $z_{1,P} = (1+\psi_0)z_{0,P} + z_0\psi_{0,P}$ in the second budget to obtain

$$\psi_{0,P} = \frac{(-NX_{1,z})(1+\psi_0)(-NX_{0,P})}{(-NX_{0,z} + V_{-1}R_0^B/z_0)[(-NX_{1,z})z_0 + rV_0]} > 0 \tag{74}$$

The repayment equation also gives $z_{1,P} = -rV_0\psi_{0,P}/(-NX_{1,z}) \leq 0$. This inequality is strict when $V_0 > 0$; with no foreign-owned claim in the second period, $z_{1,P} = 0$ at the binding limit. At a switching point use the feasible one-sided derivative. The sign establishes an increase in the premium; a positive level additionally requires a nonnegative starting premium. The zero-premium reference in Proposition 2 supplies that condition.

C.2 A simultaneous investor withdrawal

Allow the foreign-investor mandate to change by $dF_0 \leq 0$ at the same time as the oil price changes by $dP_0^O \geq 0$. Inherited positions, the second-period oil price and the monetary and reserve settings remain

fixed. Differentiating the stock identity and both external budgets now gives

$$dD_0 = -dF_0 + \left(-NX_{0,z} + \frac{V_{-1}R_0^B}{z_0}\right) dz_0 - NX_{0,P}\, dP_0^O \quad (75)$$

$$(-NX_{1,z})dz_1 + rV_0 d\psi_0 + r[1 + (1-s)\psi_0]dD_0 = -r(1+\psi_0)dF_0 \quad (76)$$

The last term matters: a lower mandate also reduces the direct foreign position receiving the next bond return. Use (34) and, inside capacity, $d\psi_0 = \gamma\, dD_0$. Let $\mathcal{D}$ denote the positive denominator of (73). Elimination yields

$$d\psi_0 = \frac{\gamma(1+\psi_0)}{\mathcal{D}} \left\{ (-NX_{1,z})(-NX_{0,P})\, dP_0^O + \left[-NX_{1,z} + r\left(-NX_{0,z} + \frac{V_{-1}R_0^B}{z_0}\right)\right](-dF_0) \right\} \quad (77)$$

Both shock coefficients are positive. Thus $d\psi_0 > 0$ and $dD_0 = d\psi_0/\gamma > 0$ if at least one shock is strict. At a strictly binding upper limit, set $dD_0 = 0$ in (75)–(76). The result is

$$d\psi_0 = \frac{(1+\psi_0)\left\{ (-NX_{1,z})(-NX_{0,P})\, dP_0^O + \left[-NX_{1,z} + r\left(-NX_{0,z} + \frac{V_{-1}R_0^B}{z_0}\right)\right](-dF_0) \right\}}{\left(-NX_{0,z} + \frac{V_{-1}R_0^B}{z_0}\right)\left[(-NX_{1,z})z_0 + rV_0\right]} > 0 \quad (78)$$

The statement is a local comparison at a common equilibrium. Finite responses need not add across shocks, and the import-spending signs proved for oil alone do not follow for a combined shock: withdrawal also lowers future direct foreign claims.

C.3 What makes this a two-period result

The first budget alone leaves the response of import spending undetermined. Choosing a spending response that exactly absorbs the oil bill would satisfy that single identity, but it need not satisfy repayment and household saving. Equations (71)–(72) impose those additional restrictions and determine the positive derivative in (73). Oil prices revert in the second period; the second-period allocation generally does not. There is no third-period return to a reference allocation in this finite economy.

D The welfare value of the second instrument

D.1 The two wedges and the welfare identity

Write u_t for period utility in (1). For a policy variation at fixed oil prices and export contracts, cost minimization and goods clearing imply

$$W_t\, dN_t + S_t P_t^O\, dX_{E,t} = mc_t\, dC_{H,t}$$

Using the household conditions and $dNX_t = -dz_t - P_t^O\, dX_{E,t}$ gives

$$\begin{aligned} du_t &= \nu_t\, dC_{H,t} + \lambda_t\, dz_t - \nu_t W_t\, dN_t \\ &= \nu_t(1 - mc_t)\, dC_{H,t} - \lambda_t\, dNX_t \end{aligned}$$

This separates the domestic wedge from the value of foreign resources. Holding monetary settings fixed sets $dC_{H,t} = 0$. The cost envelope then becomes

$$W_t N_{t,z} + S_t (P_t^O X_{E,t})_z = 0$$

Since $\chi = \nu_t W_t$ and $S_t \nu_t = \alpha / z_t = \lambda_t$, period utility satisfies

$$u_{t,z} = \frac{\alpha}{z_t} - \chi N_{t,z} = \lambda_t [1 + (P_t^O X_{E,t})_z] = \lambda_t (-NX_{t,z}) \tag{79}$$

Equivalently, $du_t = -\lambda_t \, dNX_t$ for this experiment. Differentiating the external budgets with respect to a reserve sale gives

$$\begin{aligned} dNX_0 &= -dZ_0 - dD_0 + V_{-1} dR_0^B, \\ dNX_1 &= r(dZ_0 + dD_0) + rV_0 d\psi_0 + r(1-s)\psi_0 dD_0 \end{aligned} \tag{80}$$

Multiply by $-\lambda_0$ and $-\beta\lambda_1$, respectively, and add. Household saving implies $\lambda_0 = (1+\psi_0)\lambda_1$ and $\beta r = 1$. For a general policy variation, restoring the domestic terms gives

$$\begin{aligned} d\mathcal{W} = \sum_{t=0}^{1} \beta^t \nu_t (1 - mc_t) \, dC_{H,t} + \lambda_1 \psi_0 (dZ_0 + s \, dD_0) \\ - \lambda_1 V_0 \, d\psi_0 - \lambda_0 V_{-1} \, dR_0^B \end{aligned}$$

At fixed monetary settings the domestic terms vanish, yielding (36). The financial term uses the same wedge $\lambda_0 - \beta r \lambda_1 = \lambda_1 \psi_0$ that appears in household saving; inherited payments and domestic intermediary earnings remain in the expression.

D.2 Proof of Proposition 3

For this proof only, use $\mathcal{D}$ for the positive denominator displayed in (73); it is an algebraic coefficient, not a new economic state. The assumption $V_{-1} \geq 0$ ensures $-NX_{0,z} + V_{-1} R_0^B / z_0 > 0$, while $V_0 \geq 0$ and $1 + \psi_0 > 0$ ensure positivity of the remaining denominator terms. The external budgets and the financial wedge imply

$$\begin{aligned} D_{0,Z} &= -1 + (-NX_{0,z} + V_{-1} R_0^B / z_0) z_{0,Z}, \\ z_{1,Z} &= (1+\psi_0) z_{0,Z} + z_0 \psi_{0,Z}, \\ (-NX_{1,z}) z_{1,Z} + rV_0 \psi_{0,Z} + r[1 + (1-s)\psi_0] D_{0,Z} + r &= 0 \end{aligned} \tag{81}$$

On the interior branch $\psi_{0,Z} = \gamma D_{0,Z}$. Solving gives

$$\begin{aligned} D_{0,Z} &= -\frac{(-NX_{1,z})(1+\psi_0) + r(-NX_{0,z} + V_{-1} R_0^B / z_0)}{\mathcal{D}} < 0, \\ z_{0,Z} &= \frac{\gamma(-NX_{1,z}) z_0 + r\gamma V_0 + r(1-s)\psi_0}{\mathcal{D}}, \\ z_{1,Z} &= (1+\psi_0) z_{0,Z} + \gamma z_0 D_{0,Z} \end{aligned} \tag{82}$$

Let $\mathcal{B}$ denote the left side minus the right side of (37), solely to collect the algebra in this proof. Substitution into (79) gives

$$\mathcal{W}_Z = \frac{\alpha}{z_0 \mathcal{D}(1+\psi_0)} \{\gamma \mathcal{B} + (1-s)\psi_0(1+\psi_0)[-rNX_{0,z} - NX_{1,z}]\} \tag{83}$$

Under the proposition's assumptions the denominator is positive and the second term in braces is nonnegative. Condition (37) makes the first term strictly positive.

At a strictly binding upper limit, $D_{0,Z} = 0$, and (81) instead gives

$$\begin{aligned} z_{0,Z} &= \frac{1}{-NX_{0,z} + V_{-1}R_0^B/z_0} > 0, \\ \psi_{0,Z} &= -\frac{r(-NX_{0,z} + V_{-1}R_0^B/z_0) + (-NX_{1,z})(1+\psi_0)}{(-NX_{0,z} + V_{-1}R_0^B/z_0)[(-NX_{1,z})z_0 + rV_0]} < 0 \end{aligned} \tag{84}$$

The resulting welfare derivative is

$$\mathcal{W}_Z = \frac{\alpha \mathcal{B}}{z_0(-NX_{0,z} + V_{-1}R_0^B/z_0)(1+\psi_0)[(-NX_{1,z})z_0 + rV_0]} > 0 \tag{85}$$

Regularity ensures a locally continuous private response; room under the policy and portfolio constraints makes a sufficiently small sale feasible. It raises welfare while keeping the monetary-only optimum's monetary settings fixed. The best feasible joint plan can do at least as well as that improved allocation. This establishes a strict improvement without requiring a globally concave joint-policy problem or a zero premium at its optimum.

The proof applies when the non-FXI inequalities permit the stated direction. In particular, domestic bond holdings must remain positive, the central bank must have room to acquire bonds, and an unchanged interest rate must satisfy the common bound. At a switching point the appropriate feasible one-sided derivative replaces the strict-branch calculation.

E Household and firm choices with staggered prices

E.1 Household derivatives

The household budget is (40), with $P_t C_t = P_{H,t}C_{H,t} + S_t(C_{M,t} + P_t^O C_{E,t})$ at the expenditure-minimizing demands. Profits Π_t^F include export-contract income and net out the lump-sum financing of the price-setting subsidy; that subsidy still affects each firm's price choice. The last term includes the predetermined foreign income and gross domestic intermediary income. If usable capital sK is assigned to intermediaries, household nontraded foreign wealth is $\bar{G} - sK$ and intermediary distributions include $s(r-1)K$. Adding the two cancels this allocation of capital and gives the term displayed above. Domestic fees and their rebates cancel as well. There is no household first-order condition for $\bar{G}$.

Adding the separate central-bank and fiscal budgets gives (54). Substitute that transfer and bond clearing into (40). The resulting aggregate equation is (50). The predetermined foreign position earns interest and is rolled over; it is not liquidated at the end of the numerical policy window. Let $\mathcal{R}_t^H$ be the right-hand side of (40) minus its left-hand side. For the choices

$$\{C_{H,t}, C_{M,t}, C_{E,t}, N_t, B_t^S, H_t\}_{t \geq 0},$$

the Lagrangian is

$$\mathfrak{L}^H = \sum_{t\geq 0} \beta^t \left[u(C_t) - \chi_N \frac{N_t^{1+\varphi}}{1+\varphi} + \nu_t \mathcal{R}_t^H + \zeta_t^H H_t \right], \qquad \zeta_t^H \geq 0 \tag{86}$$

Intermediary distributions and the fee rebate are taken as given by the household. Their sum is gross domestic intermediary income; no domestic fee is separately deducted from utility or aggregate resources. For $j = H, M, E$, write $C_{C_j,t}$ for the derivative of the nested consumption aggregate with respect to that good. The first-order conditions are

$$\begin{aligned} C_t^{-\sigma} C_{C_H,t} &= \nu_t P_{H,t}, & C_t^{-\sigma} C_{C_M,t} &= \nu_t S_t, & C_t^{-\sigma} C_{C_E,t} &= \nu_t S_t P_t^O, \\ \chi_N N_t^{\varphi} &= \nu_t W_t, & \nu_t Q_t^S &= \beta \nu_{t+1}, & \nu_t Q_t^B &= \beta \nu_{t+1}(1 + \delta_B Q_{t+1}^B) + \zeta_t^H, \\ \zeta_t^H &\geq 0, & H_t &\geq 0, & \zeta_t^H H_t &= 0 \end{aligned} \tag{87}$$

For example, $C_{C_H} = (1-\alpha)^{1/\eta}(C/C_H)^{1/\eta}$; imported goods use the chain rule through C_F. Dividing the two inner demands gives their relative expenditure. Substituting into the binding CES aggregate then gives

$$\begin{aligned} C_{M,t} &= (1-\omega)(S_t/P_{F,t})^{-\vartheta} C_{F,t}, & C_{E,t} &= \omega(S_t P_t^O/P_{F,t})^{-\vartheta} C_{F,t}, \\ P_{F,t}^{1-\vartheta} &= (1-\omega) S_t^{1-\vartheta} + \omega (S_t P_t^O)^{1-\vartheta} \end{aligned} \tag{88}$$

The outer nest gives (41)–(42) by the same steps. Homogeneity ensures that $P_t C_t = P_{H,t} C_{H,t} + S_t C_{M,t} + S_t P_t^O C_{E,t}$ at these demands. Continuous limits apply at unit elasticities. Concavity, the linear budget, no-Ponzi condition and transversality make the household conditions sufficient.

At $H_t > 0$, the two bond Euler equations give $I_t Q_t^B = 1 + \delta_B Q_{t+1}^B$, hence $R_{t+1}^B = I_t S_t / S_{t+1}$. Multiplying $\nu_t = \beta I_t \nu_{t+1}$ by S_t yields (55). Iterating the long-bond equation gives (47) under the no-bubble condition. The household corner retains the inequality multiplier in (87).

E.2 Assembly, factor choice, and Calvo prices

A competitive assembler minimizes expenditure on varieties subject to delivering the CES aggregate. Its first-order condition is

$$P_{H,t}(j) = P_{H,t} Y_t^{1/\epsilon_p} Y_t(j)^{-1/\epsilon_p}$$

Thus $Y_t(j) = [P_{H,t}(j)/P_{H,t}]^{-\epsilon_p} Y_t$, and substitution gives $P_{H,t}^{1-\epsilon_p} = \int_0^1 P_{H,t}(j)^{1-\epsilon_p} dj$. Total expenditure equals $P_{H,t} Y_t$, so assembly profit is zero.

Given demand $Y_t(j)$, producer j chooses its inputs to solve

$$\min_{N_t(j), X_{E,t}(j) \geq 0} \{ W_t N_t(j) + P_{E,t} X_{E,t}(j) \} \quad \text{subject to (44)} \tag{89}$$

The Lagrangian is input expenditure plus $P_{H,t} mc_t$ times demanded output minus the CES output in (44). Positive factor prices make the constraint bind. Its conditions are

$$W_t = P_{H,t} mc_t \frac{\partial Y_t(j)}{\partial N_t(j)}, \qquad P_{E,t} = P_{H,t} mc_t \frac{\partial Y_t(j)}{\partial X_{E,t}(j)} \tag{90}$$

Differentiating the production function gives

$$\frac{\partial Y_t(j)/\partial N_t(j)}{\partial Y_t(j)/\partial X_{E,t}(j)} = \frac{1-\mu}{\mu}\left[\frac{N_t(j)}{X_{E,t}(j)}\right]^{-1/\nu_E}$$

Dividing the factor conditions gives the input ratio in (45). The production function and that ratio yield

$$\begin{aligned} N_t(j) &= Y_t(j)A^{\nu_E-1}(1-\mu)^{\nu_E}\left(\frac{P_{H,t}mc_t}{W_t}\right)^{\nu_E}, \\ X_{E,t}(j) &= Y_t(j)A^{\nu_E-1}\mu^{\nu_E}\left(\frac{P_{H,t}mc_t}{P_{E,t}}\right)^{\nu_E} \end{aligned} \tag{91}$$

Their expenditure equals $P_{H,t}mc_tY_t(j)$, proving the unit-cost formula. Since every firm uses the same input ratio, constant returns imply that aggregate inputs produce $\int_0^1 Y_t(j)dj = \Delta_t^p Y_t$. Substituting those inputs into the same CES expression gives (46).

A resetting producer chooses one price $P_{H,t}^{\#} > 0$ to maximize

$$\sum_{k\geq 0}(\beta\theta_p)^k \nu_{t+k}[(1+\bar{\varsigma}^p)P_{H,t}^{\#} - P_{H,t+k}mc_{t+k}]\left(\frac{P_{H,t}^{\#}}{P_{H,t+k}}\right)^{-\epsilon_p} Y_{t+k} \tag{92}$$

Demand and cost minimization have already been substituted, so there are no remaining equality constraints in this single-price choice. For $P = P_{H,t}^{\#}$, a summand's derivative, apart from its positive weight, is

$$P^{-\epsilon_p-1}P_{H,t+k}^{\epsilon_p}Y_{t+k}[-(\epsilon_p-1)(1+\bar{\varsigma}^p)P + \epsilon_p P_{H,t+k}mc_{t+k}]$$

Summing and setting the derivative to zero gives

$$P_{H,t}^{\#} = \frac{\epsilon_p}{(\epsilon_p-1)(1+\bar{\varsigma}^p)}\frac{\sum_{k\geq 0}(\beta\theta_p)^k\nu_{t+k}mc_{t+k}P_{H,t+k}^{1+\epsilon_p}Y_{t+k}}{\sum_{k\geq 0}(\beta\theta_p)^k\nu_{t+k}P_{H,t+k}^{\epsilon_p}Y_{t+k}} \tag{93}$$

The derivative is positive below this price and negative above it; it is the unique positive maximizer. The subsidy makes the leading factor one. Divide the numerator sum by $P_{H,t}^{\epsilon_p}$ to define K_t^p and the denominator sum by $P_{H,t}^{\epsilon_p-1}$ to define L_t^p. Separating current from future profits and splitting the price index and dispersion integrals between resetting and nonresetting firms gives

$$\begin{aligned} K_t^p &= \nu_t P_{H,t}mc_tY_t + \beta\theta_p\Pi_{H,t+1}^{\epsilon_p}K_{t+1}^p, \\ L_t^p &= \nu_t P_{H,t}Y_t + \beta\theta_p\Pi_{H,t+1}^{\epsilon_p-1}L_{t+1}^p, \\ p_t^{\#} &= K_t^p/L_t^p, \\ 1 &= \theta_p\Pi_{H,t}^{\epsilon_p-1} + (1-\theta_p)(p_t^{\#})^{1-\epsilon_p}, \\ \Delta_t^p &= \theta_p\Pi_{H,t}^{\epsilon_p}\Delta_{t-1}^p + (1-\theta_p)(p_t^{\#})^{-\epsilon_p} \end{aligned} \tag{94}$$

The auxiliary sums K_t^p, L_t^p are present values of price-setting terms, not assets. Their discounted definitions select the nonexplosive recursions.

F Welfare decomposition across horizons

F.1 Production and the domestic wedge

Consider a policy variation at a given oil-price path and investor mandate. Fixed export quantities imply $dY_t = dC_{H,t}$. Constant returns and cost minimization give

$$\frac{W_t}{P_{H,t}}\, dN_t + \frac{S_t P_t^O}{P_{H,t}}\, dX_{E,t} = mc_t \left(\Delta_t^p\, dC_{H,t} + Y_t\, d\Delta_t^p\right) \tag{95}$$

Household goods conditions value domestic consumption at $\nu_t P_{H,t}$ and imported spending at $\lambda_t = S_t \nu_t$. Labor optimality values dN_t at $\nu_t W_t$, whether disutility is linear or convex. Thus

$$du_t = \nu_t P_{H,t}\, dC_{H,t} + \lambda_t\, d(C_{M,t} + P_t^O C_{E,t}) - \nu_t W_t\, dN_t$$

Use (95), and substitute

$$dNX_t = -d(C_{M,t} + P_t^O C_{E,t}) - P_t^O\, dX_{E,t}$$

This proves (57). The identity does not assume optimal policy, zero inflation, or a slack intermediary limit. It describes a policy variation at fixed shocks; differentiating the oil shock itself would add a direct oil-price term.

F.2 External payments and the financial wedge

For periods $0, \ldots, T$, summing the differentiated external budgets gives

$$\begin{aligned}\sum_{t=0}^{T} \beta^t \lambda_t\, dNX_t = &\sum_{t=0}^{T-1} \beta^t (\lambda_t - \beta r \lambda_{t+1})\, dA_t + \beta^T \lambda_T\, dA_T + \lambda_0 V_{-1}\, dR_0^B \\ &+ \sum_{t=0}^{T-1} \beta^{t+1} r \lambda_{t+1} [V_t\, d\psi_t + (1-s)\psi_t\, dD_t]\end{aligned} \tag{96}$$

Initial portfolios are held fixed, so $dA_{-1} = dV_{-1} = 0$; their first-period return can change. In later periods $dR_{t+1}^B = r\, d\psi_t$ and $dV_t = (1-s) dD_t$. With fixed mandates and $\bar{G}$, $dA_t = dR_t - dD_t$. Substitute these identities and (55) into (96), then use (57). The exact welfare derivative is

$$\begin{aligned}d\mathcal{W}_{0:T} = &\sum_{t=0}^{T} \beta^t \nu_t P_{H,t} [(1 - mc_t \Delta_t^p) dC_{H,t} - mc_t Y_t\, d\Delta_t^p] \\ &+ \sum_{t=0}^{T-1} \beta^{t+1} r \lambda_{t+1} [\psi_t(-dR_t + s\, dD_t) - V_t\, d\psi_t] \\ &- \lambda_0 V_{-1}\, dR_0^B - \beta^T \lambda_T\, dA_T\end{aligned} \tag{97}$$

The first line contains domestic production and price dispersion. The second contains financing and foreign payments. The last line contains inherited valuation and the boundary position. In the infinite horizon the boundary disappears under the corresponding marginal transversality condition. In a finite numerical prefix it must be retained and combined with continuation welfare.

For the two-period economy, set $T = 1$, $dA_1 = 0$, $P_{H,t} = 1$ and $\Delta_t^p = 1$. Since $-dR_0 = dZ_0$ and $\beta r = 1$, equation (97) first gives the general two-wedge decomposition in Appendix D. Fixing the two

monetary settings then makes $dC_{H,0} = dC_{H,1} = 0$ and yields (36). In the infinite-horizon economy, $-dR_t = \sum_{j=0}^{t} dZ_j$: a reserve sale changes future reserve stocks, not only its current flow.

F.3 A feasible intervention test

At a constrained monetary-only optimum, the derivative along an admissible monetary variation cannot be positive. Adding FXI admits additional directions. If one of them has a positive complete derivative in (97), a sufficiently small feasible intervention raises welfare. The criterion uses all terms, including inherited valuation, price dispersion and portfolios carried forward; it does not identify the premium alone with a welfare loss.

I evaluate such a direction at the quantitative monetary-only solutions by retaining their first 24 interest rates and subsequent Taylor rule, and introducing a small first-quarter reserve sale. At $s = 0, 0.5, 1$, the exact derivatives are positive for both shock configurations. Six successively smaller interventions in each case reproduce the derivative in independently solved private equilibria. The welfare identity agrees with direct utility differentiation to 4×10^{-14}, and the extrapolated finite-difference derivatives agree to 6×10^{-7} in derivative units. These are checks of the mechanism; the welfare gains in the main tables are obtained from the separately optimized policy plans.

Under these fixed-rate variations, inherited valuation contributes positively and price dispersion negatively to the total derivative. A fixed rate in the infinite-horizon economy with Calvo pricing does not hold both periods' domestic spending fixed, as the finite monetary-direction test does. The positive net derivative is therefore the relevant connection between the two models, not an assertion that every component has the same sign.

G Numerical implementation and policy comparisons

G.1 Solution, continuation and welfare

For each shock and ownership share, I solve the nonlinear commitment problem over the 24-quarter control window with 144 explicit equilibrium quarters. Candidate plans allow different quarters of binding and slack intermediary capacity. For each candidate pattern, the equations and inequalities retain the original household, firm, external and public budgets. Neighboring capacity patterns and multiple starting plans are compared before retaining the highest verified welfare among the candidates. This establishes numerical local optimality and policy feasibility; it is not a global-concavity theorem.

The central bank's controls are then imposed in an independent private-equilibrium solution. Rates follow the stated Taylor rule after quarter 24 and intervention stops, while the economy carries forward its actual assets and price distribution. The distant continuation uses the stable transition and its local curvature around the reference allocation. The original nonlinear equilibrium is replayed over 420 and 840 quarters before evaluating welfare. No reserve stock, bond position or consumption allocation is reset at the policy-window boundary.

The checks cover all forty paths: five ownership shares, two shocks, and Taylor, monetary-only, immediate joint and delayed joint policies. The maximum equilibrium residual is 3.8×10^{-14}, and the maximum separate household cash-budget residual is 1.4×10^{-15}. Policy bounds and positive household and central-bank bond holdings are checked at the equilibrium transaction prices. Extending the private replay from 420 to 840 quarters changes welfare by at most 2.2×10^{-8} CE bp and the paired cost of delay by at most 7.6×10^{-9} CE bp. Domestic fee rebates are included consistently in both the event solution and continuation welfare.

The monetary-coordinate equivalence is checked independently. With log consumption, nominal expenditure satisfies

$$I_t = \frac{P_{t+1}C_{t+1}}{\beta P_t C_t}$$

Replacing the first 24 rate coordinates with these exact expressions, while retaining every rate bound and the same continuation rule, reproduces private allocations within 9.4×10^{-13}. Calling the rate implementation monetary policy therefore leaves the existing quantitative policy set unchanged.

G.2 The value of intervening in the first quarter

Let $v(c)$ be the highest welfare attainable when the first reserve sale is restricted by $Z_0 \leq c$, while later decisions are optimized. Because opening reserves are at their ceiling, the first operation cannot be a reserve purchase. Consequently

$$v(0) = \mathcal{W}_{\text{delayed}}, \qquad v(\bar{Z}) = \mathcal{W}_{\text{joint}}$$

The feasible sets are nested. On a regular optimizing branch the multiplier on $c - Z_0$ is $v'(c)$; if v is absolutely continuous, the utility cost of delay is

$$\mathcal{W}_{\text{joint}} - \mathcal{W}_{\text{delayed}} = \int_0^{\bar{Z}} v'(c)\, dc$$

This is a property of the quantitative policy menus, not a separate three-period economy. The positive cost of delay in the combined experiment means that excluding first-quarter reserve trades removes useful feasible allocations even though the central bank can promise later intervention.

G.3 Ownership and external payments

Keeping allocations and initial total positions fixed, the derivative of the external-budget residual with respect to domestic ownership is

$$\frac{\partial}{\partial s}\{rA_{t-1} + NX_t - (R_t^B - r)[F_{t-1} + (1-s)D_{t-1}] - A_t\} = (R_t^B - r)D_{t-1}$$

The quadratic domestic fee has no separate utility derivative because its receipts are rebated. Ownership changes the foreign incidence of inherited returns and subsequent intermediary earnings. It also changes the equilibrium allocation when policies are reoptimized. Thus this accounting derivative alone is not a theorem that every FXI gain decreases with domestic ownership. The monotone cost-of-delay result reported in Table 7 comes from solving the full policy comparison at each ownership share.

G.4 Reported quantities and provenance

Consumption and exchange-rate responses are percentage changes from the common stationary reference, with the domestic price level accumulated from domestic inflation. Annualized rate and inflation changes are four times their quarterly log changes, in percentage points. The premium is the quarterly holding return relative to r, in basis points. Reserve sales, intermediary positions and transaction flows are divided by fixed reference value added Y^v; their denominator is not current GDP.

Every table and quantitative figure is generated from the same forty saved nonlinear private-equilibrium paths and their discounted utility. The financing decomposition reconstructs net exports,

current bond prices and exchange rates independently and verifies the signed stock-flow identity before rounding. The numerical appendix reports the adopted rebated-fee economy throughout; comparisons from specifications with a direct domestic utility cost are not mixed into these tables.

H Production coefficients and reporting units

The production function in (8) is the conventional CES form. A benchmark-input representation is an equivalent parameterization, widely used when calibrating input shares; see Klump et al. (2011). To match reference factor prices and quantities, choose

$$\mu = \frac{\bar{P}_E \bar{X}_E^{1/\nu_E}}{\bar{W}\bar{N}^{1/\nu_E} + \bar{P}_E \bar{X}_E^{1/\nu_E}},$$
$$A = \frac{\bar{Y}}{[(1-\mu)\bar{N}^{(\nu_E-1)/\nu_E} + \mu \bar{X}_E^{(\nu_E-1)/\nu_E}]^{\nu_E/(\nu_E-1)}} \tag{98}$$

The first equality matches the marginal-product ratio to $\bar{W}/\bar{P}_E$; the second matches gross output. The resulting cost share is

$$\frac{\bar{P}_E \bar{X}_E}{\bar{W}\bar{N} + \bar{P}_E \bar{X}_E} = \frac{\mu^{\nu_E} \bar{P}_E^{1-\nu_E}}{(1-\mu)^{\nu_E} \bar{W}^{1-\nu_E} + \mu^{\nu_E} \bar{P}_E^{1-\nu_E}}$$

With $\bar{Y} = \bar{P}_E = 1$, $\bar{X}_E = 0.03265728$, $\bar{N} = 0.99962689$, $\bar{W} = 0.96770378$ and $\nu_E = 0.4$, this gives $\mu = 0.0001993087$ and $A = 1.02262745$. These coefficients reproduce the same technology, input demands and marginal cost as the benchmark-input representation. Reference inputs are calibration observations, not additional states or constraints on production.

The model uses foreign-good units for $F_t, D_t, R_t, A_t, \bar{G}, Z_t$ and NX_t. Numerical storage and reported tables use these quantities divided by Y^v, the fixed reference value added defined in Section 7. Multiplying a stored position by Y^v recovers the model quantity. In particular, a slope of 0.001 with a GDP-normalized intermediary position corresponds to $\gamma = 0.001/Y^v$ in (13). All stock and intervention bounds are converted by the same factor; rates, premiums, choices and welfare do not change. This conversion keeps a change in current GDP from mechanically altering a reported financing flow.